CHAPTER 9
MULTIVALENT LOGICS

9.1. In this chapter I sketch a truth-functional tetravalent approach to multivalence.

I speak of a sketch because the matter is vast. Subsequent chapters (in particular Chapter 15 on variables and Chapter 16 on indexicality) will better enlighten some here apodictic assumptions. Anyhow, in order to simplify all that can be simplified without adulterating the main discourse, I agree

- to consider only propositions adduced by sentences of the atomic form subject-predicate;
- to presuppose the semantic competence of the interpreter, who then can distinguish between sortally correct and sortally incorrect propositions.

Furthermore, in order not to be charged with my own admission (Introduction: *too many quotations? they are simply an awkward attempt to mask frightening cultural gaps*) many quotations have been censored.

9.2. The hard difficulties entailed by truth-functional approaches to trivalence induced many authors to abandon them; I claim that overcoming a basic inadequacy affecting the current trivalent logics is sufficient to overcoming such difficulties. First of all, let me distinguish between two families of trivalences, respectively called "cognitive" and "sortal".

9.2.1. The cognitive trivalence, via Lukasiewicz, ascends to Aristotle himself. It refers its third alethic value to some informational gap; while Kripke (1975, p.87) claims that the third alethic value *may be interpreted as "possibility",* Kleene (1974, p.333) claims (more clearly, in my opinion) that it *means only the absence of information*.

When we speak of bivalence or multivalences, we are speaking of alethic values (of their number); and any alethic value results from a collation. Now ascertaining the cognitive impossibility of concluding a collation with a pro-collation (truth) or with an anti-collation (falsity) is ascertaining the undecidability of the piece of information (proposition) under scrutiny (§8.4). Usually (§10.1) this undecidability depends on the poorness of the statute which, as such, does not allow the verification of the unambiguous proposition to collate; yet it may also depend on some ambiguity of the proposition under collation. In other words, the *absence of information* which usually affects the institutive stage, may affect the propositive stage. For instance, since we know that Ava is a silent but mentally troubled lady, the undecidability of

Ava is quiet

does not depend on our ignorance about Ava's personality, but on the ambiguity of *quiet*. Nevertheless both an incompleteness of the statute or a fuzziness (Fine 1975) of the proposition entail a lack of information, therefore they can be associated in a classification focused on the distinction between cognitive and sortal trivalences.

9.2.2. The sortal trivalence refers its third alethic value to propositions vitiated by some semantic improperness. For instance while Martin (1968, p.325) writes *there is a distinguishable class of odd sentences whose oddity results from a kind of category-incorrectness or non-fitting of subject and predicate*, Thomason (1972, p 209) calls "deviant" the sortally incorrect 'sentences' (propositions) and writes: *the deviation arises from the application of the predicate to something of the wrong sort*. The point is clear.

9.2.3. No reasonable confusion is possible between undecidability and improperness. The former needs more information, therefore it presupposes the sortal correctness, otherwise merely semantic considerations would be sufficient to refuse the proposition under scrutiny. At most we can remark that the criterion of interpretative collaboration (the principle of charity) induces us to privilege a proper reading wherever it is possible.

9.3. The mentioned difficulties (§9.2) are well known. In fact (Thomason 1972, p.229) *intuition seems to pull in opposite directions*: and the presence of incompatible alethic tables for the same connective, as for instance Kleene's strong and weak ones (Kleene 1974, p. 334) is the direct consequence of such contradictory *pulls*.

With an eye toward the examples below I agree that

(9.i) Ava is married and Ava is pleasant

are true propositions,

(9.ii) Ava is quiet and Ava is unquiet

are undecidable propositions,

(9.iii) Ava is unmarried and Ava is unpleasant

are false propositions,

(9.iv) Ava is demonstrable and Ava is undemonstrable

are improper propositions. Furthermore, for the sake of concision, in the examples below pairs of conjoined propositions are reduced to pairs of conjoined predicates et cetera.

9.4. First of all, I treat a strictly cognitive trivalence (where improper propositions are excluded).While AT_1 (where the current and more compact tabulation is adopted)

$h_1\&h_2$	T	U	F
T	T	U	F
U	U	U	F
F	F	F	F

is the unproblematic table for the conjunction, AT_2

h	$\sim h$
T	F
U	U
F	T

is the unproblematic table for the (oppositive) negation (symbolized by "~").To avoid misundestandings, alethic tables, generally speaking, say that if certain basic propositions have certain alethic values (if the collation between certain basic propositions and the statute of reference gives certain results), then the proposition obtained by operating on the previous ones through the logical connection under scrutiny have a certain alethic value.

So for instance the *"F"* occurring in the first column, third row of AT_1 says that where

$k\&\ h_1 = \perp$

and

$k\&\ h_2 = k$

then

$k\&\ h_1\&\ h_2 = \perp$.

9.4.1. Both AT_1 and AT_2 are unproblematic in the sense that our intuition does not offer room for any tenable alternative in the assignation of their alethic values. For instance, under (9.iii) and (9.ii),

(9.v) Ava is unmarried and quiet

must be false, though we do not know whether Ava is actually quiet, because to know that she is not unmarried is sufficient to conclude that she is not unmarried-and-quiet.

The same structure of AT_1 validates the already remarked hierarchy (§8.16) among alethic values: in cognitive trivalence the dominance scale for conjunction $F>U>T$ complies with the standard interpretation according to which a conjunction takes the value of the 'least true' conjunct.

9.4.2. Of course AT_1 and AT_2 allow the compilation of the alethic table for whatever propositional connective in cognitive trivalence. In particular AT_3

$h_1 \vee h_2$	T	U	F
T	T	T	T
U	T	U	U
F	T	U	F

is the alethic table for the inclusive disjunction (in cognitive trivalence). A passage to emphasize is that though AT_3 has been obtained by a mechanical (a dull?) application of AT_1 and AT_2 to

(9.vi) not(not ... and not ...)

(that is to the canonical definition of inclusive disjunction), the same AT_3 can also be obtained through the merely intuitive way telling us that an inclusive disjunction takes the value of its 'most true' disjunct. So, under (9.iii) and (9.ii)

(9.vii) Ava is unmarried or quiet

must be undecidable; and really, since, contrary to (9.v), if Ava were quiet (9.vii) would be true but if Ava were unquiet (9.vii) would be false, not knowing whether Ava is quiet or unquiet is not knowing whether (9.vii) is true or false.

Exactly because in this context truth prevails over undecidability and undecidability prevails over falsity, we can say that the dominance scale for inclusive disjunction in cognitive trivalence is $T>U>F$.

9.5. A strictly sortal trivalence (where no cognitional gaps are admitted) requires a much more meticulous analysis. The logic of improperness, at least from the social viewpoint, is a risky matter; if we insist on submitting improper propositions to our honourable neighbours, their probable reaction is not to debate our sortal theory, but to assure us we are absolutely right and to disappear forever.

This notwithstanding, until we deal with conjunctions, no problem arises. In fact AT_4

$h_1\&h_2$	T	F	I
T	T	F	I
F	F	F	I
I	I	I	I

complies perfectly with our intuition. For instance, under (9.i) and (9.iv),
(9.viii) Ava is married and demonstrable
is manifestly improper (*T&I=I*, concisely written) because under (9.iv) no married-and-demonstrable lady can exist.

The only (and quite superficial) perplexity might concern *F&I=I*. Why not *F&I=F?*. The answer is easy. If
Ava is unmarried-and-demonstrable
were false, its (oppositive) negation ought to be true; then, as
Ava is not unmarried-and-demonstrable
is
(9.ix) Ava is unmarried-and-undemonstrable, or married-and-demonstrable, or married-and-undemonstrable
the same (9.ix) ought to be true: is it? I read (9.ix) as a manifestly improper proposition.

Another argument supporting *F&I=I* runs as follows. In general, while a false proposition like for instance
371293 is prime
is refused on the grounds of a verification, a sortally incorrect proposition like for instance
371293 is vegetarian
is refused only on the grounds of semantic considerations; therefore, since a proposition like
2599051 is prime-and-vegetarian
does not need any verification in order to be refused (that is; since the arithmetical characteristics of 2599051 are of no moment) the proposition is improper.

So, as our intuition does not offer room for any tenable alternative in the assignation of alethic values, AT_4 is the unproblematic table for conjunction in sortal trivalence.

9.5.1. The ticklish problem concerns the table for negation(s). In order to argue in the ordinary language about this matter we have to fix an informal reading for exclusive negation; thus I propose to read the exclusive negation of "*Pa*" as "*a* cannot at all be *P*" so making "cannot at all be" a declaration of improperness.

Denying a proposition is rejecting its truth. Therefore, until the denied proposition is proper, denying it is affirming the truth of its opposite; but if the denied proposition is improper, its opposite too is improper, and no improper proposition can be true. So for instance,
(9.x) Ava is not unmarried
and
Ava is married
are equivalent, but
(9.xi) Ava is not undemonstrable
and
(9.xii) Ava is demonstrable
are far from being equivalent: in fact while (9.xii) is surely improper, in (9.xi) the exclusive reading of "is not" (that is the reading making it a synonym of "cannot at all be"), makes true the same (9.xi). But just because both in (9.x) and in (9.xi) the denials are performed by a "not", as soon as improper propositions are admitted, either we accept "~" as an intrinsically ambiguous symbol, or we must introduce a second symbol ("¬", say) for exclusive negations. The decision is obvious: ambiguity is the worst enemy of logic.

Unfortunately there is so faint a consent about what exactly an exclusive negation is, that some authors refuse tout court its right to exist. In this sense establishing the respective alethics might reveal itself a rather questionable task. For instance someone could claim that only improper propositions can be denied exclusively or that the exclusive negation of a proper proposition is equivalent to its oppositive negation and so on. These different opinions more that symptoms of a puzzling situation, are choices born just by the mentioned vagueness of *exclusive negation*. Is there some puzzle regarding Ava's quietness, or simply the necessity to achieve a better (heuristic) definition of *quiet*?

My claim is that in sortal trivalence oppositive and exclusive negations are ruled by AT_5

h	$\sim h$	$\neg h$
T	F	F
F	T	F
I	I	T

where the only debatable assignations concern the third column (concisely written, $\sim T=F$ and $\sim F=T$ follow directly from AT_2 and $\sim I=I$ is banal).

First of all I emphasize that it is impossible to define "¬" in terms of the primitive "& " and "~". The simplest way to ascertain this impossibility is to scrutinize the last rows of AT_4 and AT_5. Since in AT_4 *"I"* is dominant and in AT_5 $\sim I=I$, no combination of conjunctions and oppositive negations can lead to a result different from I; on the contrary in AT_5 $\neg I = T$. Such an assignation follows directly from the same notion of exclusive negation (it is true that Ava cannot at all be demonstrable). On the contrary $\neg T=F$ and $\neg F=F$ are intuitively supported by the evidence that, as *improper* is exactly the opposite of *proper*, the exclusive negation of a proper proposition is false (not improper); for instance, stating that Ava cannot at all be (un)married is false, not improper.

9.6. The basic idea of a tetravalent approach to trivalence is simple: since proper and improper statements can be freely conjoined (as for instance in (9.viii)) and since both proper and improper statements can be oppositively or exclusively denied, the two proposed theorizations can be unified through a table for conjunction and a table for negations where both U and I occur. The task is unequivocally accomplished on the only grounds of the above tables, that is on the only grounds of the different rank U and I have in the respective hierarchies. While AT_1 tells us that in conjunction U, so to say, is F-recessive, AT_4 tells us that F is I-recessive.

In other words. The tetravalent approach follows from a paradigm according to which a proposition is

(9.xiii) - either P (proper, sortally correct) or I (improper, sortally incorrect)
- whether proper, either D (decidable) or U (undecidable)
- whether decidable (therefore proper), either T (true) or F (false).

Of course (9.xiii) is not the only possible paradigm. In my opinion it is the best one yet, in order not to be involved in superfluous quarrels, I simply claim that it is the reasonable paradigm here followed.

So AT6

$h_1\&h_2$	T	U	F	I
T	T	U	F	I
U	U	U	F	I
F	F	F	F	I
I	I	I	I	I

And AT7

h	$\sim h$	$\neg h$
T	F	F
U	U	F
F	T	F
I	I	T

are the tetravalent alethic tables for conjunction and negations. Let me emphasize that no arbitrary intervention affects the editing of AT_6 and AT_7 ($\neg U=F$, follows from the same consideration leading to $\neg T=F$ and $\neg F=F$: for instance, quite independently of the undecidability of Ava's (un)quietness, to state that Ava cannot at all be quiet is false).

9.6.1. Realizing the different rank of U and I is understanding the root of the puzzles affecting the usual approaches to trivalence: if we pretend to identify U and I in one only value (the value expressed by Thomason's asterisk, say) we introduce an incurable conflict of dominance.

This passage may be even more evident in AT8

$h1\vee h2$	T	U	F	I
T	T	T	T	T
U	T	U	U	U
F	T	U	F	F
I	T	U	F	I

that is in the table for inclusive disjunction where (obviously because of its duality with conjunction) the aforementioned $I>F>U>T$ becomes $T>U>F>I$.

9.6.1.1. Indeed AT_8 has been compiled by a (dull) application of the previous tables to the canonical scheme

(9.xiv) $\sim(\sim...\&\sim...)$

yet its values could also be obtained by the intuitive suggestion, evidence showing that such compilation is not affected by any puzzling situation.

9.7. From AT_6 and AT_7 we can derive any other connective. The set of combinatory connections, owing to the presence of two different primitive symbols for negation, is very numerous. For instance the scheme not (not *h'* and not *h"*)

bears eight different symbolizations ((9.xiv) is nothing but one of them, precisely that where the three "not"s are read as oppositive negations).

The sake of frankness compels me to confess that I have analyzed only a little fragment of the whole set; what I can assure is that, as far as I verified it, the suggestions of our intuition are satisfied by the values resulting from the (dull?) application of AT_6 and AT_7.

In order to show that the very reason why our intuition seems sometimes to pull in opposite directions is nothing but the ambiguity of the problem under scrutiny I dwell on a specific scheme.

9.8. The scheme concerns the propositional connective

(9.xv) if *h'* then *h"*

that is the connective which in the truth functional approach to conditionals corresponds to

(9.xvi) not*(h'* and not *h")*

(I remind the reader that a systematic theorization of conditionals, that is a wider approach to (9.xv), will be proposed in Chapter 14).

The scheme is here applied to some contingent examples.

9.8.1. Under (9.i) and (9.iv)

(9.xvii) If Ava is married, then she is demonstrable

instances a conditional with a true protasis and an improper apodosis; so

(9.xviii) not(Ava is married and not demonstrable)

is its reading in terms of conjunction and negations. Yet three incompatible arguments lead to three different alethic values for (9.xvii), or indifferently for its equivalent (9.xviii).

Argument I. So that a conditional be proper, both its protasis and its apodocis must be proper; but the apodosis of (9.xvii) is improper, therefore the same conditional is improper.

Argument II. Since no person can be demonstrable or undemostrable, Ava cannot be married and not demonstrable, just what (9.xviii) states. Therefore (9.xvii) is true.

Argument III. Since Ava is married, and no person is demonstrable, (9.xviii) denies the conjunction of two true propositions. Therefore (9.xvii) is false

9.8.2. Analogously, on the grounds of (9.ii)

(9.xix) If Ava is married, then she is quiet

that is

(9.xx) not(Ava is married and not quiet)

instances a conditional with a true protasis and an undecidable apodosis.

Of course, new conclusions impose.

Argument IV. The apodosis of (9.xix) is undecidable; yet if it were true, the conditional too would be true, and if the apodosis were false, the conditional too would be false. Therefore (9.xix) is undecidable.

Argument V. Since Ava is married and may be unquiet, we cannot state that Ava is not married-and-unquiet. Therefore (9.xix) is false.

Arguments VI. Since we cannot exclude that Ava is quiet, we cannot state that she is married and not quiet. Therefore (9.xix) is true.

9.8.3. Finally, again under (9.i) and (9.iv)

(9.xxi) If Ava is demonstrable then she is married

that is

(9.xxii) not(Ava is demonstrable and unmarried)

instances a conditional with an improper protasis and a true apodosis. The respective conclusions follow.

Argument VII. Surely (9.xxi) cannot be false, since its apodosis is true.

Argument VIII. So that a conditional be proper, both its protasis and its apodosis must be proper (Argument I); but the protasis of (9.xxi) is improper, therefore the same (9.xxi) is improper.

Argument IX. Since no person can be demonstrable, Ava cannot be demonstrable and unmarried, just what (9.xxii) states; therefore (9.xxi) is true.

9.9. Since in (9.xvi) "not" occurs twice, tetravalence can distinguish among four interpretations of (9.xvi) (that is, so to say, four sorts of conditionals) respectively listed in columns (7), (8), (9) and (10) of AT_9

(1) h'	(2) h''	(3) ;h''	(4) $\neg h''$	(5) h'&;h''	(6) $h'\&\neg h''$	(7) $\sim(h'$&;$h'')$	(8) $\neg(h'\&\sim h'')$	(9) $\sim(h'\&\neg h'')$	(10) $\neg(h'\&\neg h'')$
T	T	F	F	F	F	T	F	T	F
T	U	U	F	U	F	U	F	T	F
T	F	T	F	T	F	F	F	T	F
T	I	I	T	I	T	I	T	F	F
U	T	F	F	F	F	T	F	T	F
U	U	U	F	U	F	U	F	T	F
U	F	T	F	U	F	U	F	T	F
U	I	I	T	I	U	I	T	U	F
F	T	F	F	F	F	T	F	T	F
F	U	U	F	F	F	T	F	T	F
F	F	T	F	F	F	T	F	T	F
F	I	I	T	I	F	I	T	T	F
I	T	F	F	I	I	I	T	I	T
I	U	U	F	I	I	I	T	I	T
I	F	T	F	I	I	I	T	I	T
I	I	I	T	I	I	I	T	I	T

(the size of AT_9 aims at inducing the reader to enlarge the analysis).

Once emphasized that the alethic values occurring in AT_9 have been obtained in the dullest way, that is through a mechanical application of AT_6 and AT_7 to (9.xvi), I comment briefly the nine arguments above.

9.9.1. The assumptions of the first three arguments correspond to the fourth row of AT_9 (true protasis, improper apodosis). Argument I is valid iff the two negations occurring in (9.xvi) are oppositive, since the exclusive negation of an improper proposition is proper. In particular, while the oppositive negation of

Ava is married and undemostrable

is improper, its exclusive negation is true. In fact

(9.xxiii) ~(married-and-undemonstrable)

is

married-and-demonstrable or unmarried-and-undemostrable or unmarried-and-demonstrable

that is the (improper) disjunction of three improper conjunctions; on the contrary

(9.xxiv) ¬(married-and-undemonstrable)

leads to

Ava cannot at all be married-and-undemonstrable

that is to an unquestionable truth (under the agreed reading of "cannot at all be")

So, just as the *"I"* occurring in the fourth row of the column (7) corresponds to (9.xxiii), the *"T"* occurring in the fourth row of the column (8) corresponds to (9.xxiv). Furthermore, as

(9.xxv) $h'\&\neg h''$

(Ava is married and cannot at all be undemonstrable) is true, both its inclusive and exclusive negations (columns (9) and (10) respectively) are false. Therefore there is no puzzle between the outcomes of the mechanical procedure and the conclusions suggested by an analysis grounded upon the intuitive evidence.

For the sake of concision I abridge the following comments which can be analogously argued.

9.9.2. The arguments from IV to VI (true protasis, undecidable apodosis) corresponds to the second row of AT_9. For instance Argument VI corresponds to column (9), and actually if we read exclusively the second "not" of (9.xx), since it is false that Ava is not at all unquiet, the conjunction is false, therefore its negation is true. Analogously Argument IV corresponds to column (7) et cetera.

9.9.3. The last three arguments correspond to the thirteenth row (improper protasis, true apodosis).

Argument VII is verified by the evidence that no *"F"* occurs in the last four columns of such a row.

Argument VIII corresponds to column (7), or indifferently to column (9), since the improperness of h' makes improper both (9.xxv) and

(9.xxvi) $h'\&\sim h''$

therefore both their oppositive negations.

Argument IX corresponds to column (8) or indifferently to column (10) since the truth of the apodosis makes false both its oppositive and exclusive negations, so making (9.xxv) and (9.xxvi) equivalent.

9.10. In conclusion, as soon as we free the truth functional approach from abusive assumptions and undue restrictions, tetravalence, besides avoiding any puzzle, opens wider horizons to our intuition.