CHAPTER 8
ALETHICS

8.1. The formal treatment of alethics could proceed quite independently of any representation. Yet, since ® helps both the exposition and the understanding of the matter, I will make a large use of diagrams. Of course such diagrams can be applied to whatever universe of reference where the statute allows a partition of the possibility space, that is a codification of the alternatives. Yet, for the sake of simplicity, I will mainly reason about the example sketched in §6.11 (the slider on the rail). Then a generic ®-diagram represents the basic statute $k°$ (concerning the eight segments of the tract) by partitioning the circle in eight virgin sectors and the acquirements k', k'' et cetera by shading the sectors corresponding to the segments of the tract precluded by such acquirements (for the sake of concision, we can only consider a single acquirement).

Our fundamental problem is representing an hypothesis and assigning an alethic value (a 'truth value') to it. I emphasize that *alethic value* must be intended in its widest acceptation, according to which not only *true* and *false*, but also *probable*, *decidable* et cetera are alethic values. The link among such notions is evident. For instance a piece of information h is k-true iff $P(h|k)=1$ and is k-false iff $P(h|k)=0$. Therefore the intrinsically relational nature of *probable* is the intrinsically relational nature of *true* et cetera. Without a reference to a statute, all alethic predicates are senseless.

8.1.1. In order not to waste time with analyses of too scarce an interest, incoherent statutes will be neglected (that is, formally: $\sim(k \supset h \& \sim h)$ is a presupposed condition). I recall §6.2: the existence of a coherent statute does not imply the ontological existence of the universe it describes. We can reason about Polyphemus exactly because the objects of logic are pieces of information quite independently on their eventual fictitiousness. Anyhow the privileged role we must recognize to the actual statute; will be treated in Chapter 13.

8.2. An alethic procedure is essentially an informational collation which, as such, can be analysed in the institutive, in the propositive and in the properly collative stages.

The institutive stage consists in the assumption of a statute k ($k=k°\&k'$) that is of an arbitrary informational endowment constituting the basic element of the collation.

The propositive stage consists in the assumption of a hypothesis h, that is of an arbitrary piece of information constituting the element to collate with the basic element.

The collative stage consists in the comparison between h and k. The various alethic predicates correspond to the different results of this comparison.

8.3. Although both an acquirement and a hypothesis are pieces of information, they play an opposite role in any alethic procedure.

Let me spend few informal words about such an opposition. This academic hall is crowded by teachers and students. All of them, after all, are human beings, therefore, till the discourse concerns generically the human beings crowding this hall, one only sort of individual variables is sufficient. But, since the role of teachers and students, as for the examinations in course, is opposite; once the discourse involves examinations, the strictly complete symbolic endowment to reason on the universe constituted by the persons crowding the hall ought to list three sorts of individual variables; for instance "x" ranging over the subset of teachers, "y" ranging over the subset of students and "z" ranging over the whole set. We could even accept to renounce "z" (replacing it by a disjunction) but we could never accept further renounces, since a symbolic endowment listing only one sort of variables would mutilate our same expressive power. Analogously, with reference to pieces of information, as soon as we deal with collations, we need at least two sorts of variables as *"k"* (for cognitions) and *"h"* (for hypotheses). Correspondingly in ® we need two sorts of marks. and I agree that in ® cognitions are represented by shadings and hypotheses by hatchings.

Therefore the elementary representation of an alethic procedure can be performed by two ®-diagrams. Both of them start from the virgin circle representing $k°$: in the first (institutive) diagram the sectors representing the alternatives precluded by k' are shaded; in the second (propositive) diagram the sectors representing the alternatives precluded by h are hatched. The collative stage is realized by comparing the relations between shaded field and hatched field. For the sake of simplicity, once agreed that shadings and hatchings can overlap, the two diagrams can be unified, thus helping the comparison.

8,4. Let h be a hypothesis concerning a statute k. We say

a) that h is k-true (symbolically: $T_k(h)$) iff h does not preclude any k-free alternative
b) that h is k-false (symbolically: $F_k(h)$) iff $\sim h$ is k-true, that is if h precludes all the k-free alternatives
c) that h is k-decidable (symbolically: $D_k(h)$) iff h is either k-true or k-false
d) that h is k-undecidable (symbolically: $U_k(h)$) iff h is neither k-true nor k-false

e) that k is h-exhaustive iff h is k-decidable (§6.4.3)
f) that k is h-incomplete iff h is k-undecidable.

8.4.1. In §10.1 the theme concerning the choice of "undecidable" in order to adduce the above agreed piece of information will be deepened.

8.4.2. The notion of h-exhaustiveness (of h-incompleteness) can be strengthened by agreeing that a statute k is absolutely exhaustive (absolutely incomplete) iff it is h-exhaustive (h-incomplete) for every h concerning its possibility space.

8.5. The definitions of §8.4 are not affected by any arbitrariness: they are dictated by previous assumptions and by the usual meanings of alethic predicates. In fact, owing to the link between *truth* and *probability* (§8.1: a piece of information h is k-true iff $P(h|k)=1$ and is k-false iff $P(h|k)=0$),

h is k-true

implies

$\mu_\kappa(h)= \mu_\kappa(k)$
$k\&h=k$
$k\supset h$

and

h is k-false

implies

$\mu_\kappa(h)= 0$
$k\&h=\perp$
$k\supset\sim h$.

Once expressed in terms of measures the definitions

$U_k(h) = (0< \mu_\kappa(h)< \mu_\kappa(k))$
$D_k(h) = (\sim(\sim(\mu_\kappa(h)=\mu_\kappa(k)\&\sim(\mu_\kappa(h)=0)))$
$T_k(h) = (\mu_\kappa(h)= \mu_\kappa(k))$
$F_k(h) = (\mu_\kappa(h)= 0)$

evidence immediately some intuitive alethic relations. For instance

that the opposite of a k-undecidable hypothesis is k-undecidable, too
that the opposite of a k-decidable and k-true hypothesis is a k-decidable and k-false one
that for every coherent statute $T_k(\emptyset)$ and $F_k(\perp)$

et cetera.

Let me insist. Alethics is an intrinsically relational doctrine because the truth (or falsity et cetera) of a hypothesis results from its collation with another information (the statute). And the informational approach evidences the intrinsically relational character of alethic predicates. In this sense *k-true* (*k-false* et cetera) is the correct notion by which *true* (*false* et cetera) must always be replaced.

8.6. The representation is immediate. So while the hatched field representing a k-true hypothesis must respect every k-virgin sector (as $\mu_\kappa(h)= \mu_\kappa(k)$), the hatched field representing a k-false hypothesis must involve every k-virgin sector and the hatched field representing a k-undecidable hypothesis must involve some but not every k-virgin sectors. Therefore undecidableness entails at least two k-virgin sectors, that is (obviously) the non-exhaustiveness of the statute (undecidableness follows from some kind of ignorance). The problem of verifying an undecidable hypothesis is the problem of acquiring new cognitions, so that the increased shaded field covers either the hatched one (thus making true the hypothesis under scrutiny) or the non-hatched one (thus making it false).

8.6.1. I do not show in detail that every formal interdependence among the various alethic predicates as, for instance,

$T_k(h) = F_k(\sim h)$,
$U_k(h) = (\sim T_k(h)\ \&\ \sim F_k(h))$
$D_k(h) = \sim(\sim T_k(h)\ \&\ \sim F_k(h))$

is adequately and unambiguously represented in ®.

For instance the absolute exhaustiveness is represented by a diagram where only one sector is virgin; in fact whatever complementary bipartition of the circle is such that exactly one of its two fields falls into the previously shaded one, therefore whatever hypothesis is decidable. Analogously the absolute incompleteness is represented by a completely virgin circle.

8.6.1.1. A pedantry. While in §8.6.1 the decidability is defined through the inclusive disjunction

$\sim(\sim T_k(h)\ \&\ \sim F_k(h))$

in §8.4 it is explained through a partitive disjunction (either true or false). The inaccuracy is only apparent, because the prejudicial condition of coherence entails

$$\sim(T_k(h) \ \&\ F_k(h))$$

that is the exclusive component of the partitive disjunction.

8.6.2. Let me insist. In ® the various alethic predicates correspond to precise diagrammatic situations whose essential discriminating factor is the 'topological' relation between the k-virgin and the h-hatched fields. Therefore the possible and reciprocally incompatible results of a collation are

I) the hatched field does not involve the k-virgin field (that is: the whole hatched field falls into the shaded field)
II) the hatched field involves the whole k-virgin field (and, eventually, a part of the shaded field)
III) the hatched field involves only a part of the k-virgin field (and, eventually, a part of the shaded field).

The case I represents a k-true, the case II represents a k-false and the case III represents a k-undecidable hypothesis (of course the case III entails an incomplete statute, because a virgin field constituted by only one sector cannot be partially hatched).

8.6.3. The example I am about to analyse starts from the statute $k=k°\&k'$ got by adding the acquirement k'

the slider is in the last quarter of the tract

to our basic $k°$. Then Figure 8.0

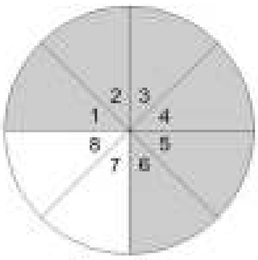

Figure 8.0

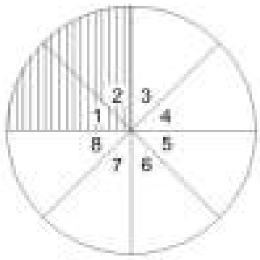

Figure 8.1

represents k. With reference to it,

the slider is not in the first quarter of the tract

(h_1 represented in Figure 8.1)

the slider is in the first half of the tract

(h_2 represented in Figure 8.2)

the slider is in an odd segment

(h_3 represented in Figure 8.3)

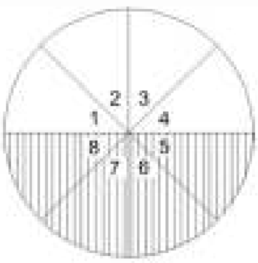

Figure 8.2

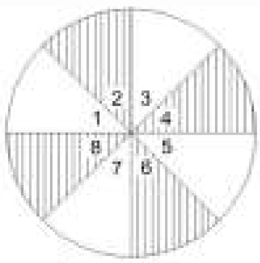

Figure 8.3

are the three different hypotheses under alethic scrutiny.

Once recalled that the k-measure of a h is represented by the area of the $k\&h$-virgin field, we can diagrammatically infer that $T_k(h_1)$, that $F_k(h_2)$ and that $U_k(h_3)$. In fact

$$\mu_\kappa(k\&h_1)= \mu_\kappa(k)$$

(both the virgin sectors of Figure 8.0, that is the sectors 7 and 8, are virgin in Figure 8.1 too)

$$\mu_\kappa(k\&h_2)= 0$$

(both the virgin sectors of Figure 8.0 are hatched in Figure 8.2)

$$0< \mu_\kappa(k\&h_3)< \mu_\kappa(k)$$

(only one virgin sectors of Figure 8.0 is hatched in Figure 8.3).

Of course the probabilistic values resulting from the areal ratios, that is $P(h_1|k)=1$, $P(h_2|k)=0$, $P(h_3|k)=1/2$, correspond to our intuitive suggestions.

8.7. A momentous theme concerns the formulation adopted in §7.1 in order to present a system of axioms. First of all I wish to avoid a possible equivocation; the momentousness does not follow from the fact that those variables, instead of ranging as usual over sentences, range over propositions. In fact, for instance, once recalled that "σ" names the semantic relation and once agreed that "e" is a variable ranging over sentences so that "σe" simply means *the piece of information adduced by e*,

if σe_1 and if σe_2, then $\sigma(e_1\&e_2)$

could replace AX3 et cetera. Keeping propositions as objects of axioms overcomes even the problem concerning the oxymoron between *substitution* and *identity* (§7.2.2) for, of course, since $\sigma e_1=\sigma e_2$ does not imply $e_1=e_2$, in

if σe_1 and if $\sigma e_1=\sigma e_2$, then σe_2

we deal with two different (therefore non identical) sentences adducing the same (therefore identical) piece of information.

8.7.1. The momentousness of the theme (that is the reason why I spoke of a propaedeutic system of axioms) depends on the non-strictness of such formulations: in fact they are omissive.

Let me be meticulous, looking at the same notion of an axiom from a general informational viewpoint. An axiom assigns an alethic value to a piece of information connected in some way with pieces of information whose alethic value is presupposed. In other words, an axiom is an instrument for inferring. Of course, since alethic values depend on the statutes of reference, any inference too depends on the statutes of reference. Actually it is easy to propose inferences concerning various alethic values and various statutes. For instance

if h_1 is $k°\&k'$-undecidable, then $h_1\& h_2$ cannot be $k°$-true

is a correct inference concerning two different hypotheses and two different statutes.

The essential dependence of alethic predicates on the statute of reference is a necessary consequence of their intrinsically relational nature.

A logic for statements concerning different statutes is a very wide matter (few notes in §15 below). Here I only deal with one only statute and with true pieces of information. These agreements allow us to omit both indications, so for instance reducing an explicit (therefore a non-omissive) formulation like

if h_1 is k-true and h_2 is k-true, then $h_1\& h_2$ is k-true

or better like

(8.i) if $T_k(h_1)$ and if $T_k(h_2)$ then $T_k(h_1\&h_2)$

to an implicit formulation like

if h_1 and h_2, then $h_1\& h_2$

that is to the formulation of AX3 proposed in §7.1. Analogously

if $T_k(h_1)$ and if $T_k(h_1=h_2)$ then $T_k(h_2)$

is the explicit formulation of AX2. And so on.

Therefore any inference concerning different statutes must make explicit the various statutes of contingent reference (explicit inferences). That is: only inferences concerning one only statute can omit the respective reference (implicit inferences).

8.7.1.1. A pedantry. In order to avoid any autonymy,"&" and "~" might be replaced by their names as "*CONG*" and "*NEG*". Under this convention, for instance,

(8.ii) If $T_k(h_1)$ and if $T_k(h_2)$ then $T_k(CONG(h_1,h_2)$

is the re-formulation of (8.i) et cetera. Yet, once this passage has been emphasized, I will prefer (8.i) to (8.ii) merely in order not to go too far from the usual symbolizations.

8.8. I call "**competence condition**" the fundamental rule according to which, in an implicit inference only pieces of information belonging to the same statute can be used. The violation of the competence condition leads to potential absurdities.

8.8.1. An easy example (all new lines are metalinguistic). While the piece of information adduced by

(8.iii) Tegucigalpa is the capital of Guatemala

is k_{Ava}-true (belongs to Ava's present statute) because actually Ava thinks so, and the piece of information adduced by

(8.iv) Tegucigalpa is the capital of Nicaragua

is k_{Bob}-true because actually Bob thinks so, evidently the piece of information adduced by the conjunction of (iii) and (iv), that is the piece of information adduced by

Tegucigalpa is the capital of Guatemala and Tegucigalpa is the capital of Nicaragua

is neither k_{Ava}-true nor k_{Bob}-true (none of them thinks that Guatemala and Nicaragua have the same capital). Yet as soon as we realize that we are reasoning about two different (and incompatible) statutes, we realize that the competence condition compels us to make explicit the respective references. And indeed, since both the pieces of information adduced by

(8.v) Ava thinks that Tegucigalpa is the capital of Guatemala and by

Bob thinks that Tegucigalpa is the capital of Nicaragua

are speaker-true, AX3 assures us that the piece of information adduced by their conjunction too, that is

Ava thinks that Tegucigalpa is the capital of Guatemala and
Bob thinks that Tegucigalpa is the capital of Nicaragua

is speaker-true.

Actually, since we know that Tegucigalpa is the capital of Honduras,

(8.vi) Tegucigalpa is the capital of Honduras

states a speaker-true identity. If we use (8.vi) for a substitution in (8.v) we get

(8.vii) Ava thinks that the capital of Honduras is the capital of Guatemala

that is a sentence whose interpretation, in spite of its resemblance to (8.v), is ambiguous. In fact if we read (8.vii) as

(8.viii) the piece of information adduced by "the capital of Honduras
is the capital of Guatemala" belongs to k_{Ava}

we face a sentence adducing a false proposition (Ava does not at all think that Honduras and Guatemala have the same capital). On the contrary if we read (8.vii) as

(8.ix) Ava erroneously thinks that the town which is the real
capital of Honduras is the capital of Guatemala

we face a sentence adducing a true proposition.

And the opposite alethic values of (8.viii) and (8.ix) are the due consequences of the competence condition. In fact the falsity of (8.viii) (recycling use of a new line, what can be false is a proposition, not a sentence), the falsity of (8.viii), then, follows from the violation of the same condition: as the piece of information adduced by (8.vi) does belong to $k_{speaker}$ but, like (8.iv), does not belong to k_{Ava}, it cannot be used for a substitution within the scope of "belongs to k_{Ava}". On the contrary the truth of the piece of information adduced by (8.ix) follows from respecting the mentioned condition: in fact such an interpretation refers to $k_{speaker}$, and as both the pieces of information adduced by (8.v) and by (8.vi) belong to such a statute, the identity stated by (8.vii) can be used for a substitution in (8.v) because we are looking at Ava's beliefs from the speaker's viewpoint (that is because the statute of reference is $k_{speaker}$).

8.8.2. The chronological dimension is not involved in the example above. Yet, of course, the difference between two statutes may also depend exclusively on a difference between the temporal reference. Here is an example..

Though at $t°$ Ava thought (8.iii), at t' she realized her mistake, and consequently replaced (8.iii) with (8.vi), which then belongs to k_{Ava} too, thus legitimating the inference of (8.vii); nevertheless (8.vii) continues being false, since Ava does not at all think that Honduras and Guatemala have the same capital. But here too we are dealing with two (incompatible) statutes: $k_{Ava,t°}$ (shortly $k°$) and $k_{Ava,t'}$ (shortly k'). If we choose to reason under $k°$ we recover the above analysis, and if we choose to reason under k', we cannot use (8.ii) because under k' Ava no longer thinks that Tegucigalpa is the capital of Guatemala. What we can correctly argue is that, since

at $t°$ Ava thought that Tegucigalpa is the capital of Guatemala

belongs to k' (Ava remembers her previous erroneous belief), the competence condition makes

at $t°$Ava thought that the capital of Honduras is the capital of Guatemala

a k'-legitimate inference; and accordingly we recognize that its conclusion is k'-true.

8.8.3. The competence condition rules all pieces of information occurring in an inference, and therefore the implicative relations too. So it would be easy to contradict the Theorem of Transitivity (Chapter 6, Theor6) or even the *Modus Ponens* by an application as

if $k\supset h_1$ and if $h_1\supset h_2$ then $k\supset h_2$

and by supposing that $h_1\supset h_2$ does not hold in k. For instance (hyperlinguistic new lines)

Flipper is a dolphin

is true for Ava, but, though

dolphin implies *mammal*

is speaker-true, since Ava is unaware that dolphins are mammals,

Flipper is a mammal

is not true for Ava. Yet the necessity of respecting the competence condition makes

(8.x) If $k\supset h_1$ and $k\supset(h_1\supset h_2)$ then $k\supset h_2$

the correct formulation (which, as such, cannot be the source of the well known puzzling questions about oblique contexts). Anyhow this topic will be scrutinized in Appendix 16.

8.8.4. Even the most obvious statement as

if h then h

could be immediately contradicted by referring the two occurrences of h to two locally incompatible statutes.
In this sense contexts where one of the different statutes is the speaker's one (so calling the statute containing previously supplied or universally known pieces of information) are peculiarly insidious.

8.9. The explicit formulations allow a stricter approach to nothing less than the same Aristotelian *Non Contradictio* (*NC*) and *Tertium Non Datur* (*TND*) principles.

Their usual symbolization, that is

(8.xi) $\sim(h \& \sim h)$

for *NC* and

(8.xii) $h \vee \sim h$

for *TND,* leads to an impasse. In fact, since by definition

$h_1 \vee h_2$

is

$\sim (\sim h_1 \& \sim h_2)$

and since (Theor15)

$\sim \sim h = h$

it follows that (8.xi) and (8.xii) are equivalent (reciprocally derivable). Quite a disconcerting conclusion, indeed, at least because qualified scholars (intuitionists, for instance) accept the universal validity of *NC* but not of *TND*.

I claim that the root of the impasse is exactly the inadequateness of the quoted formulations. What *NC* and *TND* intend to establish is that two opposite pieces of information cannot be both true nor both non-true. Therefore both principles are hyperlinguistic statements whose (hyper-)information results from the attribution of an alethic predicate to some (object-)information. In this sense (8.xi) and (8.xii) are mutilating because the alethic predicates are omitted. And as soon as this undue mutilation is corrected by replacing (8.xi) and (8.xii) respectively with

(8.xiii) $\sim (T_k(h)\ \&\ T_k(\sim h))$

and with

$T_k(h)\ \vee\ T_k(\sim h))$

that is with

(8.xiv) $\sim(\sim (T_k(h)\ \&\ \sim T_k(\sim h))$

the impasse vanishes; in fact (8.xiii) and (8.xiv) are not at all equivalent. While *NC* holds both in bivalence and in trivalence (no coherent logic can accept dilemmas $|h|$ where both h and $\sim h$ are true) *TND* holds only in bivalence (trivalence rules undecidable dilemmas too, and any undecidable dilemma contradicts (8.xiv)). In other words: the simple coherence of a statute k is sufficient to exclude that both h and $\sim h$ can be k-true, but only the $|h|$-exhaustiveness of a statute k can assure that at least one of the two opposite hypotheses is k-true. Thus the reason is explained why *TND* may fail with reference to incomplete statutes: because the non-incompleteness condition limits its universal validity. And just because reality is absolutely exhaustive (the exhaustive coherence of the world we live in is recognized even by some logicians) *TND* is valid in the logic of an ideal knower.

8.9.1. The definitions of §8.4 allow the formal derivation of the conclusions above (I remind the reader that, in order not to waste time, we are reasoning under the presupposition of coherence for k).

As for *NC*. Let $T_k(h)$. By definition: $k\&h=k$. By Theor9: $k\&\sim h=\perp$. By definition: $F_k(\sim h)$. By definition: $\sim T_k(\sim h)$. Therefore *NC* is derivable without any condition of bivalence.

As for *TND*. Let $T_k(h) \vee T_k(\sim h))$, that is $\sim(\sim (T_k(h)\ \&\ \sim T_k(\sim h).)$

By *NC*: $\sim(T_k(h) \&\ T_k(\sim h))$. Therefore either $T_k(h)$ or $T_k(\sim h)$. By definition either $T_k(h)$ or $F_k(h)$.

Therefore $\sim(\sim T_k(h) \&\ \sim F_k(h))$ that is $\sim U_k(h)$*:* Ergo *TND* implies bivalence.

8.10. Also the diagrammatic counter-part of the derivations proposed in §8.9.1 is immediate (the k-coherence entails a non completely shaded circle, that is the presence of at least one virgin sector).

As for *NC*. The complement of a hatched field falling into the shaded field cannot fall into the same shaded field quite independently on the number of virgin sectors (therefore *NC* rules also non-exhaustive statutes, that is it rules both bivalence and trivalence).

As for *TND*. In order to be sure that at least one of two complementary fields falls into the shaded field, the virgin field cannot be formed by more than one sector. Therefore *TND* is valid only under exhaustive statutes, that is only in bivalence.

8.11. Two last words (referred directly to ®) about the decision of neglecting incoherent statutes. Let me consider a $k=\perp$, that is a completely shaded circle, that is a $\mu_k(k)=0$. On the one hand I could claim that every hypothesis is k–true because any hatching falls into the shaded field (here is the representation of the scholastic *ex absurdo quodlibet*). On the other hand I could also claim that every hypothesis is k–false, because no virgin sector survives to shading and hatching. The absurdity of the situation, in my opinion, legitimates only one trustworthy conclusion: that incoherence is too deceiving a topic to be analyzed in a rational way. Therefore I firmly insist in neglecting incoherent statutes.

8.12. A possible objection concerning the re-partitioning technique runs as follows.

Let us call "k°-pregnant" a hypothesis iff it concerns such a possibility space, being neither tautologic nor contradictory. While the technique by shading admits pregnant and true hypotheses, the technique by re-partitioning does not. For instance, if we reduce Figure 8.0 to a virgin circle bi-partitioned in the sectors 7 and 8, no hatching can interest only shaded sectors, therefore no pregnant hypothesis can be true.

Reply. Under the re-partitioning technique those $k°$-pregnant hypotheses are true whose hatching does not interest the virgin circle. I remind the reader (§6.13) that this technique is less complete (it adduces less information) than the technique by shading, where the precluded alternatives too are represented. In other words. Once we agree that the precluded alternatives are neither represented, we agree not to represent any true piece of information.

8.13. The following TABLE 1

	h	$\sim h$
(8.xv)	T_k	F_k
(8.xvi)	F_k	T_k
(8.xvii)	U_k	U_k

rules the alethics of negation; in TABLE 1, while (8.xv) and (8.xvi) concern bivalence, (8.xvii) concerns trivalence. A tetravalent approach to trivalence (no oxymoron) will be proposed in Chapter 9. Here I presuppose that every piece of information we deal with is sortally correct.

Diagrammatically it is evident that a trivalent logic cannot concern an absolutely exhaustive statute (if k leaves a monosectorial virgin field, either such a sector is h-hatched, and then h is k-false, or it is h-virgin, and then h is k-true). Since a trivalent logic can only depend on some lack of information, we could say that bivalence is the logic of the ideal knower (or of a human knower whose statute is exhaustive as for the hypotheses under scrutiny).

I emphasize that the compilation of TABLE 1 does not need any integrative assumption; in fact all the alethic values for $\sim h$ follow theoremically from the respective alethic values for h.

8.14. The following TABLE 2

TABLE 2	h_1	h_2	$h_1\& h_2$
(8.xviii)	T_k	T_k	T_k
(8.xix)	T_k	F_k	F_k
(8.xx)	F_k	T_k	F_k
(8.xxi)	F_k	F_k	F_k
(8.xxii)	T_k	U_k	U_k
(8.xxiii)	U_k	T_k	U_k
(8.xxiv)	F_k	U_k	F_k
(8.xxv)	U_k	F_k	F_k
(8.xxvi)	U_k	$U_k\&F_{k\&h1}$	F_k
(8.xxvii)	U_k	$U_k\&\sim F_{k\&h1}$	U_k

establishes the alethics of conjunction in its simplest version. While (8.xviii), (8.xix), (8.xx) and (8.xxi) concern bivalence, (8.xxii), (8.xxiii), (8.xxiv), (8.xxv) (8.xxvi) and (8.xxvii) concern trivalence. Also the compilation of TABLE 2 does not need any integrative assumption, since all the alethic values for $h_1\& h_2$ follow theoremically from the respective alethic values for h_1 and for h_2. Let me show this concisely.

The derivations of (8.xviii) (8.xix), (8.xx) and (8.xxi) are similar. I sketch the derivation of (8.xix): $T_k(h_1)$ ergo $k\&h_1=k$; $F_k(h_2)$ergo $k\&\sim h_2=k$. Theor7: $k\&h_1\&\sim h_2= k$. Theor10 $k\&h_1\&h_2= \perp$. By definition $F_k(h_1\&h_2)$.

The compatibility of h_1 and h_2 is implicitly assured in (8.xviii) and in (8.xxi), since two incompatible pieces of information (owing to the coherence of k) cannot be both k-true or both k-false. And the eventual incompatibility of h_1 and h_2 is of no moment in (8.xix) and (8.xx), since their conjunction would anyway be false.

The derivation of (8.xxii) runs as follows. Since $k\&h_1=k$, $\sim(k\&h_2=k)$, $\sim(k\&\sim h_2=k)$, surely $h_1\&h_2$ cannot be k-true because, if it were, then $k\&h_2=k$, contrary to $U_k(h_2)$. But neither $h_1\&h_2$ can be k-false, because if $k\&\sim(h_1\&h_2)=k$, since $k\&h_1=k$ and $h_1\&\sim(h_1\&h_2)\supset\sim h_2$, then $k\&\sim h_2=k$, contrary to $U_k(h_2)$. But $\sim T_k(h_1\&h_2)\& \sim F_k(h_1\&h_2)$ is just $U_k(h_1\&h_2)$. The incompatibility between h_1 and h_2 is an eventuality that we must reject because if they were incompatible, $T_k(h_1)$ would entail $F_k(h_2)$.

The derivation of (8.xxiii), (8.xxiv) and (8.xxv) is analogous.

On the contrary the derivation of (8.xxvi) and (8.xxvii) requires a more detailed analysis. Since $U_k(h_1)$, $h_1\&h_2$ cannot be k-true because if it were k-true $k\& h_1\&h_2 =k$, therefore $k\&h_1=k$, contrary to $U_k(h_1)$. So either $F_k(h_1\&h_2)$ or $U_k(h_1\&h_2)$. If $F_k(h_1\&h_2)$, then $(k\&(h_1\&h_2)=\perp$, then $((k\&h_1)\&h_2)=\perp)$, then $F_{k\&h1}(h_2)$. And if $\sim F_{k\&h1}(h_2)$, then $\sim((k\&h_1)\&h_2=\perp)$, then $\sim(k\&(h_1\&h_2)=k)$, then $\sim F_k(h_1\&h_2)$. But $\sim T_k(h_1\&h_2)\& \sim F_k(h_1\&h_2)$ is just $U_k(h_1\&h_2)$. This analysis corresponds to the paradigm offered by ®. In fact we have two topologically different diagrams representing an h_1 and an h_2 such that $U_k(h_1)$ and $U_k(h_2)$. In one of them the two hatchings, once joined, do not leave any virgin sector, and then $F_k(h_1\&h_2)$). In the other they do, and then $U_k(h_1\&h_2)$).

For instance, under the usual $k°$ (a rail partitioned in 8 segments et cetera), once

the slider is in the first half of the tract

is assumed as k' and

the slider is in the first quarter of the tract

is assumed as the k-undecidable h_1. if the k-undecidable h_2 is

(8.xxvi*) the slider is in the second quarter of the tract

$h_1\&h_2$ is k-false, while if the k-undecidable h_2 is

(8.xxvii*) the slider is in the segment 2

$h_1\&h_2$ is k-undecidable.

8.15. Since the alethic value of a h depends strictly on the statute of reference, in general it is impossible to infer the k_1-alethic value of a h from its k_2-alethic value, unless k_1 and k_2 are linked by some implicative relation.

With the aim of widening our theoretical perspective I propose the following TABLE 3

	h	h
(8.xxviii)	$T_{k°}$	$T_{k°\&k'}$
(8.xxix)	$F_{k°}$	$F_{k°\&k'}$
(8.xxx)	$U_{k°}$	$T_{k°\&k'} \downarrow F_{k°\&k'} \downarrow U_{k°\&k'}$
(8.xxxi)	$T_{k\text{ç}\&k'}$	$\sim F_k$
(8.xxxii)	$F_{k°\&k'}$	$\sim T_{k°}$
(8.xxxiii)	$U_{k°\&k'}$	$U_{k°}$

where the alethic values of an h are referred to a basic statute $k°$ and to a statute $k=k°\&k'$ obtained by increasing $k°$ with a cognition k'. Also the compilation of TABLE 3 does not need any integrative assumption, since et cetera. For instance (8.xxviii) ensues from the Theorem of Transitivity. In fact if $T_{k°}(h)$, then by definition $k°\supset h$, thence, as $k°\&k'\supset k°$. $k°\&k'\supset h$, that is $T_{k°\&k'}(h)$. Analogously for (8.xxix): if $F_{k°}(h)$, then by definition $k°\supset\sim h$, thence, as $k°\&k'\supset k°$, $k°\&k'\supset\sim h$, that is $F_{k°\&k'}(h)$.

Both (8.xxviii) and (8.xxix) are immediately evident in ®. For instance, as for (8.xxviii), if the hatching involves no virgin sector of $k°$, it involves no virgin sector of $k°\&k'$ whose shaded field is either the same ($k'=Ø$) or greater than $k°$.

Analogously, as for (8.xxx) if the $k°$-diagram entails that the non-shaded sectors be partially hatched, the further k'-shading is compatible with three different situations et cetera.

Not less immediate is the diagrammatic verification of (8.xxxi) (8.xxxii) and (8.xxxiii). The respective formal proofs are obtainable from the same line, here limited to (8.xxxi). $T_{k\text{ç}\&k'}\&F_k$ is contradictory because, owing to (8.xxix), $F_{k°}$ implies $F_{k°\&k'}$ which implies $\sim T_{k°\&k'}$, therefore by *Modus Tollens* $T_{k°\&k'}$ implies $\sim F_k$.

8.15.1. TABLE 3 leads directly to the Theorem of Conservation. Incrementative acquirements of a statute do not alter the truth or falsity of a hypothesis. This theorem ensues directly from (8.xxviii) and (8.xxix) and constitutes a milestone for human knowledge. In fact it assures that, once an assumed statute leads to a conclusion about the alethic value of a hypothesis, such a conclusion continues holding under every increment of the assumed statute. Of course the circumstances where we must modify our previous conclusions about te alethic value of a hypothesis abound, yet they presuppose a correction of the statute, and any correction presupposes an ablation, therefore, first of all, a decrement.

8.15.2. TABLE 3 concerns a single hypothesis. Its complete extrapolation to the case of two hypotheses would entail a too detailed tabulation. I simply emphasize that the task of compiling TABLE 4 as, for instance,

	h_1	h_2	$h_1\&h_2$
	…..	…..	…..
(8.xxxiv)	$T_{k°}$	$U_{k°\&k'}$	$U_{k°\&k'}$
	….	….	….

is only a matter of patience. So (8.xxxiv), ruling the alethic value corresponding to the conjunction of a $k°$-true h_1 with a $k°\&k'$-undecidable h_2, is derivable as follows. If $T_{k°\&k'}(h_1\&h_2)$,then $T_{k°\&k'}(h_2)$ incompatible with the presupposition $U_{k°\&k'}(h_2)$; therefore $\sim(T_{k°\&k'}(h_1\&h_2))$. If $F_{k°\&k'}(h_1\&h_2)$, since on the basis of (8.xxviii) $T_{k°}(h_1)\supset T_{k°\&k'}(h_1)$, then $F_{k°\&k'}(h_2)$, incompatible with the presupposition $U_{k°\&k'}(h_2)$. Therefore $\sim(F_{k°\&k'}(h_1\&h_2))$. But if $\sim(T_{k°\&k'}(h_1\&h_2))$ and $\sim(F_{k°\&k'}(h_1\&h_2))$ then by definition $U_{k°\&k'}(h_1\&h_2)$.

8.15.3. Far from being a fault, the complexity of the paradigm is quite favourable evidence. In fact, since each line of the above tables refers to a punctual and distinct situation, any approach leading to a less articulate paradigm would only reveal its insufficiency.

8.16. Two important theorems follow.

Theorem of Restriction:

(8.xxxv) Every k-restriction of a k-true piece of information is necessarily k-true

(of course a k-restriction is a restriction valid in k), then (8.xxxv) is nothing but (8.x), that is the correct formulation of *Modus Ponens*). The formal proof of (8.xxxv) passes through the Theorem of Transitivity, yet it is immediately derivable from TABLE 2 since $T_k(h_1)$ and $T_k(h_2)$ is the only combination of values corresponding to $T_k(h_1\&h_2)$.

Theorem of Expansion:

(8.xxxvi) Every k-expansion of a k–false piece of information is necessarily k-false

(of course a k-expansion is an expansion valid in k); (8.xxxvi) is immediately derivable from TABLE 2 since $F_k(h_1)$ or $F_k(h_2)$, are separately sufficient to imply $F_k(h_1\&h_2)$.

Corollary:

(8.xxxvii) No k-restriction of a k-undecidable piece of information can be k-false

(otherwise we could exhibit a k-undecidable expansion of a k-false piece of information, so contradicting (8.xxxvi))

On the basis of (8.xxxv), (8.xxxvi) and (8.xxxvii) we can establish a sort of alethic hierarchy among truth (wherever recessive), undecidableness (truth-dominant but falsity-recessive) and falsity (wherever dominant).

8.17. Summarising. Since the information we can infer from a certain acquirement depends also on the statute incremented by the same acquirement, the statute plays a fundamental role: therefore we must be aware of the necessity to inhibit any confusion among different statutes even where no explicit reference occurs. The puzzles affecting intentional identity contexts are born simply by a confusion among the plurality of statutes they involve (in Appendix to Chapter 16 we shall see that inhibiting any confusion is overcoming such puzzles).

Recognizing a plurality of possible statutes, obviously, is perfectly compatible with the incontestable existence of a privileged one, that is, so to say, the final appeal statute concerning the real world we live in (Chapter 13 is specifically devoted to this topic).

8.18. In Chapter 9 the well known distinction between oppositive and exclusive negation will be analyzed carefully. Here sortally incorrect propositions are neglected, and as such only oppositive negations are considered. Nevertheless an even more fundamental distinction concerning negations needs to be focalized.

Let me reason directly on ® and let me consider two complementary propositive diagrams (hatching on a virgin circle); in the former only the sector 3 is not hatched, in the latter only the sector 3 is hatched. Therefore

(8.xxxviii) the slider is in sector 3

and respectively

(8.xxxix) the slider is not in sector 3

are the sentences adducing the represented propositions h and $\sim h$.

Now let me suppose that in the institutive diagram representing the statute of reference k (shading on a virgin circle) the sector 3 is shaded, so making (8.xxxviii) k-false and (8.xxxix) k-true. Of course we can state the result of the collation between hatched and shaded fields through

(8.xxxx) *the slider is not in sector 3* is k-true

but in the usual practice, particularly where the statute of reference is unmistakable, (8.xxxx) is replaced by (8.xxxix), which then becomes an ambiguous message. In fact (I continue availing myself of ®) given the representation of a hypothesis (for instance the diagram representing (8.xxxviii)) the "not" through which we mean its complementary diagram (that is the diagram representing (8.xxxix)), is the same "not" through which we mean the (anti-collative) result of the collation between the hypothesis and the statute. Thus we fall into a projective ambiguity (that is an ambiguity involving the dialinguistic order) because a hyperlinguistic statement as (8.xxxx) may be confused with a protolinguistic statement as (8.xxxix).

The above (§8.9) critical approach to the current formulations of *NC* and *TND* is nothing but an application of the just proposed considerations. And actually this topic is strictly connected with the already denounced convention (§1.11.1): *the worst symbolic convention … is the universal habit according to which affirmation is expressed by omitting the symbol of negation*.

8.19. Just as

if $T_k(h_1)$ and $T_k(h_1 \supset h_2)$ then $T_k(h_2)$)

symbolizes the Theorem of Restriction,

if $F_k(h_1)$ and $T_k(h_2 \supset h_1)$ then $F_k(h_2)$)
symbolizes the Theorem of Expansion.

As for undecidable hypotheses

(8.xxxxi) if $U_k(h_1)$ and $T_k(h_1 \supset h_2\,)$ then $\sim F_k(h_2)$)

(in fact if $F_k(h_2)$ then since $T_k(h_1 \supset h_2)$ by Theorem of Expansion $F_k(h_1)$, contrary to $U_k(h_1)$) and

(8.xxxxii) if $U_k(h_1)$ and $T_k(h_2 \supset h_1\,)$ then $\sim T_k(h_2)$)

(in fact if $T_k(h_2)$ then since $T_k(h_2 \supset h_1)$ by Theorem of Restriction $T_k(h_1)$, contrary to $U_k(h_1)$); therefore (I recall that the identity between two pieces of information is a reciprocal implication, and as such it entails both of them)

if $U_k(h_1)$ and $T_k(h_1 = h_2\,)$ then $U_k(h_2)$)

since both (8.xxxxi) and (8.xxxxii) hold.

So, once assumed "$\aleph$" as a variable on alethic values (*true*, *false*, *(un)decidable*), we can resume the above achievements in

if $\aleph_k(h_1)$ and $T_k(h_1 = h_2)$ then $\aleph_k(h_2)$)

which tells us that the substitution of identity is an alethically conservative operation (rule of inference). In other words: contrary to *Modus Ponens* the substitution of identity does not maintain only the truth, but also the falsity and the (un)decidability.