APPENDIX TO CHAPTER 16
INTENTIONAL IDENTITY

A16.1. Here I wish to tackle (and to solve) some well known puzzles (grandiloquently: paradoxes): all of them are born by the violation of the competence condition, according to which (§8.8) an inference is valid in a statute iff **every** piece of information employed in such an inference belongs to such a statute.

And since statutes concerning distinct persons or the same person in distinct moments may differ, we can accordingly distinguish two kinds of violations: the individual and the chronological ones.

A16.2. The puzzles of intentional identity do not involve the chronological dimension, and consequently the chronological index can be omitted in the respective symbolization (so we can speak elliptically of a $k_{g,}$ instead of a $k_{g,t}$).

I tackle them making reference to Edelberg 1986; by "Ed(1)", "Ed(2)" et cetera I recall respectively the formulae (1), (2) et cetera of the mentioned paper. In order to facilitate the collations between his formulae and mine, I accept his notations even where (for instance his use of *"E"* for *"E!"*) more meticulous conventions might be preferable. So, for instance

$\mathbf{B}_{g,}[Ex(Px)]$

means that *g* (at any moment *t* interested by the reasoning under scrutiny) believes there is an *x* being *P*, or, in other words, that

Ex(Px)

is a datum (a piece of information) belonging to $k_{g,}$.

A16.2.1. I epitomize the first puzzle as follows. While (Detective) A believes Smith and Jones have been murdered by two different persons, (Detective) B believes they have been murdered by the same person. Since (p.13) A *and* B *discussed the case at length*, and since both of them ignore the specific person who murdered Smith, we can assume

murdering Smith

as a common identifying connotation.

In Edelberg's opinion (Ed(19) and Ed(20), p.13) the puzzle is that, though

(A16.i) A believes someone murdered Smith and B believes he murdered Jones

and

(A16.ii) B believes someone murdered Jones and A believes he murdered Smith

are *symmetric* (*i,e,*: though (A16.ii) is the commutation of (A16.i) and commutability is an universally accepted property of conjunction), (A16.i) is true and (A16.ii) is false. Such a commutation is even more evident if, once any existential import is dropped from quantifiers, we formalize (A16.i) in

(A16.iii) $Ex[\mathbf{B}_a(Sx) \& \mathbf{B}_b(Jx)]$

(Ed(21), p.14) and (A16.ii) in

(A16.iv) $Ex[\mathbf{B}_b(Jx) \& \mathbf{B}_a(Sx)]$

(Ed(22), ibidem). The use of the same variable both within the scope of $\mathbf{B}_a$ and of $\mathbf{B}_b$ is justified by the mentioned common identifying connotation; *x* is simply the otherwise unknown person who, both in A's and B's opinion, murdered Smith.

A16.3. My claim is that the puzzle is only a superficial fallacy caused by a mistaken choice of the antecedent converting the indexical variable "he". For the moment, let me reason in the ordinary language.

A16.4. Claiming that (A16.i) and (A16.ii) are respectively true and false presupposes that in both of them the anaphoric function succeeds in individuating a well specified referent for "he". So, with reference to (A16.i),

(A16.v) who is he?

or in other words

(A16.vi) who is the person B believes murdered Jones?

is our basic question. Once left out of consideration that B suspects A, the possible answers to (A16.v) are three, and precisely

(A16.vii) he who actually murdered Smith

(A16.viii) he whom A believes murdered Smith

(A16.ix) he whom B believes murdered Smith.

In order to prove that (A16.viii) is the only correct answer, let me consider an interim similar context where

- Smith has been actually murdered by his secretary Charlie
- A believes that Smith has been murdered by his barber Donald
- B believes that Smith has been murdered by his pedicure Edward.

Under this context

(A16.x) A believes Donald murdered Smith and B believes he murdered Jones
is unquestionably false. In fact ((A16.x) tells us that B believes Donald murdered Jones; and by hypothesis Donald, far from being (A16.vii) or (A16.ix), is (A16.viii). This conclusion can anyway be validated by supposing that A, instead of suspecting Donald, suspects Smith's partner Fred; under this new context

A believes Fred murdered Smith and B believes he murdered Jones

is true if and only if B believes that Jones has been murdered by Fred.

Therefore, since the person B believes murdered Jones must be the person A believes murdered Smith, the anaphoric function states unequivocally that the antecedent of "he" is "he whom A believes murdered Smith". And the same conclusion holds perfectly with reference to (A16.i), because the difference between (A16.i) and (A16.x), is simply that in the former case A is not in condition to give a precise identity to his suspect; a difference concerning A's opinion which obviously cannot modify the anaphoric function.

A16.4.1. Exactly as (A16.viii) is the antecedent of the "he" occurring in (A16.i),

(A16.xi) he whom B believes murdered Jones

is the antecedent of the "he" occurring in (A16.ii).

Realizing that, exactly as

(A16.i*) A believes someone murdered Smith
and B believes he whom A believes murdered Smith murdered Jones

is the only strict periphrasis of (A16.i),

(A16.ii*) B believes someone murdered Jones
and A believes he whom B believes murdered Jones murdered Smith

is the only strict periphrasis of (A16.ii), is overcoming the puzzle. In fact the condition assuring the truth of (A16.i*), that is the identity between the person A believes murdered Smith and the person B believes murdered Jones, is perfectly reflected in the condition assuring the truth of (A16.ii*), that is the identity between the person B believes murdered Jones and the person A believes murdered Smith (by hypothesis both of them identify themselves with the person B believes murdered Smith).

A16.4.2. Formally and generally speaking, wherever a

(A16.xii) Px

occurs within the scope of a **B** as for instance in

(A16.xiii) $\mathbf{B}_a(Px)$

the x we are speaking of is not

(A16.xiv) $\iota x(Px)$

(the individual being P) but

(A16.xv) $\iota x[\mathbf{B}_a(Px)]$

(the individual A believes being P) because the speaker, far from stating that x is P, states that A believes that x is P. The speaker might even dissent from ((A16.xii) and nevertheless he could legitimately state ((A16.xiii). This trivial conclusion can be also supported by comparing

(A16.xvi) $Px \,\&\, \mathbf{B}_b(Qx)$

with

(A16.xvii) $\mathbf{B}_a(Px) \,\&\, \mathbf{B}_b(Qx)$

and by remarking that, exactly as ((A16.xiv) is the antecedent of the second individual variable occurring in ((A16.xvi), (A16.xv) is the antecedent of the second individual variable occurring in ((A16.xvii).

Therefore the strict periphrases of ((A16.xvii) are

$\mathbf{B}_a(Px) \,\&\, \mathbf{B}_b(Q(\iota x[\mathbf{B}_a(Px)])$

and

(A16.xviii) $\mathbf{B}_b(Qx) \,\&\, \mathbf{B}_a(P(\iota x[\mathbf{B}_b(Qx)])$

respectively. The root of Edelberg's untenable claim is mistaking

(A16.xix) $\mathbf{B}_b(Qx) \,\&\, \mathbf{B}_a(P(\iota x[\mathbf{B}_a(Qx)])$

for ((A16.xviii), that is mistaking A's and B's (incompatible) statutes apropos of *he who murdered Jones*.

A16.4.3. The intriguing collateral question

why does the truth of (A16.ii) need a subtle analysis while the truth of (A16.i) is evident?

can be immediately answered; while mistaking (A16.viii) for (A16.ix) is inconsequential on the truth of (A16.i) because A's and B's statutes agree apropos of Smith's murderer, mistaking

(A16.xx) he whom A believes murdered Jones

for (A16.xi), that is, symbolically, mistaking (A16.xviii) for (A16.xix), upsets the truth value of (A16.ii) because A's and B's statutes disagree apropos of Jones's murderer. And mistaking (A16.xx) for (A16.xi) is suggested by a hasty reading of (A16.ii) where "… someone murdered Jones … A believes he murdered Smith" actually occurs.

A16.5. Geach's original puzzle concerns the (seeming) impossibility of theorizing a *de re* or a *de dicto* use of pronouns in intentional identity contexts. Since from the informational viewpoint such a distinction is of no moment because the ontological dimension is not necessary for the accomplishment of a semantic course (cannot we properly speak of Polyphemus?), I propose an over-schematized but more direct version of the same puzzle, where the problem concerning the real existence of witches can be completely neglected (we speak of witches exactly as we could speak of Cyclopes).

Example 1. Hob and Nob agree that the witch Hob thinks blighted Bob's mare is the witch Nob thinks killed Cob's sow.

Example 2. Nob is unaware of Hob's existence. Yet both of them know that Peter thinks a pestiferous witch roamed over the hills. Hob and Nob think (independently, of course) their witch is Peter's one.

The puzzle runs as follows. Undoubtedly

(A16.xxi) Hob thinks a witch blighted Bob's mare and Nob thinks she killed Cob's sow

is true with reference to both examples; therefore, since, on the grounds of the conclusion achieved in §4 above,

((A16.xxii) the witch Hob thinks blighted Bob's mare

is the antecedent of the "she" occurring in ((A16.xxi),

(A16.xxiii) Hob thinks a witch blighted Bob's mare
and Nob thinks the witch Hob thinks blighted Bob's mare killed Cob's sow

is obtained from ((A16.xxi) simply by replacing "she" with its antecedent ((A16.xxii). Yet while ((A16.xxiii) is true with reference to Example 1, it cannot be true with reference to Example 2, where Nob is unaware even of Hob's existence.

A16.5.1. This reasoning, once more, violates the competence condition. While in Example 1 what the speaker knows, then in particular the identity

(A16.xxiv) the witch Hob thinks blighted Bob's mare is the witch Nob thinks killed Cob's sow

is also known by Nob and as such can be used in any inference concerning Nob's beliefs, in Example 2, by hypothesis (p.2: *Nob has no belief at all about Hob or about Bob's mare*) the speaker and Nob have different statutes; in particular (A16.xxiv) is an identity known only by the speaker; therefore it cannot be used in any inference concerning Nob's beliefs. With reference to Example 2, the only legitimate antecedent for the "she" occurring in ((A16.xxi) is

the witch Peter thinks roamed over the hills

because

(A16.xxv) the witch Peter thinks roamed over the hills is the witch Nob thinks killed Cob's sow

is the only identity Nob knows. And actually

(A16.xxvi) Hob thinks a witch blighted Bob's mare
and Nob thinks the witch Peter thinks roamed over the hills killed Cob's sow

is an unobjectionable inference. In this sense the simple respect of the competence condition prevents us from deriving any puzzle.

In other words. With reference to Example 1, so to write, (A16.xxi) is speaker-true and Nob-true, but with reference to Example 2 (A16.xxi) is only speaker-true.

A16.5.2. The formal approach to Geach's puzzle follows plainly from the considerations of §A16.4.2. The competence condition is clear: only data (in particular, only identities) occurring within the scope of a "$\mathbf{B}_{g,}$" can be used for any derivation within the scope of the same "$\mathbf{B}_g$". So while

$$[\mathbf{B}_{g^{*}}(x=y)\ \&\ \mathbf{B}_g(Px)]\supset \mathbf{B}_g(Py)$$

is a misleading rule, and in particular (where g^* is the speaker)

$$[(x=y))\ \&\ \mathbf{B}_g(Px)]\supset \mathbf{B}_g(Py)$$

is a peculiarly misleading rule,

$$[\mathbf{B}_g(x=y)\ \&\ \mathbf{B}_g(Px)]\supset \mathbf{B}_g(Py)$$

is the right one.

A16.5.3. In order to sketch formally few details of the reasoning proposed in §A16.5 1, a simplification is possible; in fact, once assumed that the individual variables range directly over witches, the omnipresent predicate "to be a witch" can be omitted. Under this assumption

$$\mathbf{B}_h[Ex(Bx)\ \&\ Ey(Cy)\&(y=x)]]\ \&\ \mathbf{B}_n[Ex(Bx)\ \&\ Ey(Cy)\&(y=x)]$$

symbolizes Example 1; then, by double substitution of identity,

$$\mathbf{B}_h[Ex(Bx\ \&\ Cx)]\ \&\ \mathbf{B}_n[Ex(Bx\ \&\ Cx)]$$

(the double substitution is correct because "$y=x$" occurs both in the scope of "$\mathbf{B}_h$" and of "$\mathbf{B}_n$", that is because both Hob and Nob are aware of the identity between their witches).

Analogously the symbolization of Example 2 is

$$\mathbf{B}_p[Ez(Rz)]\ \&\ \mathbf{B}_h[Ex(Bx)\ \&\ Ez(Rz)\&(x=z))]\ \&\ \mathbf{B}_n[Ey(Cy)\ \&\ Ez(Rz)\&(y=z))]$$

where "$\mathbf{B}_p$" symbolizes "Peter thinks that" and "R" is the predicate for the pestiferous witch roaming over the hills. If we look at the question from the speaker's viewpoint, where

(A16.xxvii) $\iota z(\mathbf{B}_p(Rz)) = \iota x(\mathbf{B}_h(Bx))$

(the speaker knows that the witch Peter thinks roamed over the hills is the witch Hob thinks blighted Bob's mare), and where

(A16.xxviii) $\iota z(\mathbf{B}_p(Rz)) = \iota y(\mathbf{B}_n(Cy))$

(the speaker knows that the witch Peter thinks roamed over the hills is the witch Nob thinks killed Cob's sow), we can immediately derive

$$\iota y(\mathbf{B}_n(Cy)) = \iota x(\mathbf{B}_h(Bx))$$

(the witch Hob thinks blighted Bob's mare is the witch Nob thinks killed Cob's sow), therefore we can derive ((A16.xxiii); nevertheless no puzzle arises since we are reasoning under the speaker's viewpoint, and from such a viewpoint ((A16.xxiii) is perfectly true because, though Nob unaware, his witch is Hob's witch.

If on the contrary we look at the matter from Nob's viewpoint, we can put ((A16.xxviii) within the scope of "$\mathbf{B}_n$" (Nob knows that his witch is the witch Peter thinks roamed over the hills) but we cannot put ((A16.xxvii) within the scope of "$\mathbf{B}_n$" (Nob does not know that the witch Peter thinks roamed over the hills is the witch Hob thinks blighted Bob's mare), therefore the derivation of

$$\iota x(\mathbf{B}_h (Bx))] = \mathbf{B}_n[\iota y(\mathbf{B}_n(Cy))$$

violates the competence comdition. Briefly; since the identity ((A16.xxvii) cannot occur within the scope of "$\mathbf{B}_n$", it cannot be used for a substitution within the scope of such an operator.

In conclusion. From the speaker viewpoint ((A16.xxiii) can be correctly derived, yet from such a viewpoint no puzzle arises because it is true. In order to make ((A16.xxiii) false, we must read it from Nob' viewpoint, but then it is incorrectly derived. So, once we realize that such a falsity is not born by the illegitimate reading of the pronoun as a pronoun of laziness, but by the violation of the competence condition, Geach's puzzle vanishes.

A16.6. The root of the impasse concerning Edelberg's Example 7 (p.18), in spite of the more intricate context, is the same; therefore its solution can be achieved through the above analysis. Shortly his claim (p.17) according to which *on its most natural reading* Ed(26) *is true* and *on its most natural reading* Ed(27) *is false* is untenable. In fact the only reading under which

Ed(26) A thinks someone murdered the mayor and
B thinks he murdered the commissioner

is true can be the speaker's reading (A does not know anything about the commissioner and his death; B does not know anything about the mayor and his death). But under the speaker's reading also

Ed(27) B thinks someone murdered the commissioner
and A thinks he murdered the mayor

is true because all the four following identities

the person B thinks murdered the commissioner is the person B thinks shot Jones
the person B thinks shot Jones is the person B thinks shot Smith
the person B thinks shot Smith is the person A thinks shot Smith
the person A thinks shot Smith is the person A thinks murdered the mayor

are known by the speaker (they all are explicit assumptions of Example 7) and as such can legitimately be used for substitutions of identity concerning $k_{speaker}$.

A16.6.1. As for the intriguing question

why does Ed(27) need a subtle analysis while Ed(26) does not?

I simply remind the reader of §A16.4.3 above.

A16.7. Also the well known Surprise Text paradox (Binkley 1968; J.H. Halpern, Y.Moses 1986) is born by a violation of the competence condition, yet such a violation concerns the chronological dimension.

I abridge the paradox as follows. A teacher announced to his private pupil Eve that on exactly one of the next two lesson days he would give her a test; but it would be a surprise test in the sense that on the evening before the test she would not know that the test would take place the next day. Eve charges the teacher with incoherence; in fact, she argues

- the test cannot take place the second day for in this case on the evening of the first day she would expect it
- the test cannot take place in the first day for, having already eliminated the second day, she would expect it.

A16.7.1. In order to formalize the situation I assume "d_i" for "the test takes place day *i*" and

$$d_1 \dot{\vee} d_2$$

for

$$\sim(\sim d_1 \& \sim d_2) \;\&\; \sim(d_1 \& d_2)$$

I recall that "$\dot{\vee}$" is the symbol for the partitive disjunction XOR, conjunction of the inclusive and the exclusive disjunctions).

Let $k°$ be Eve's initial knowledge concerning the possible dates of the test; since

$$k° = (d_1 \dot{\vee} d_2)$$

if we suppose that the test did not take place on the first day,

$k^1 = k°\&{\sim}d_1$

is her knowledge at the evening of the day 1; then manifestly the acquirement

${\sim}d_1$

makes $k°$ and k^1 two different statutes.

The condition of surprise

$S(d_j)$

is by definition

(A16.xxix) ${\sim}(k^{j-1} \supset d_j)$

(where of course k^{j-1} is Eve's statute at the evening before the j-day).

A16.7.2. The paradoxical flavour of the situation depends on the incompatibility between the conclusion drawn by Eve (no surprise is possible) and the actual component of surprise entailed by the lack of information (d_1? d_2?) affecting the announcement. But such an incompatibility is born by the ambiguity of ((A16.xxix); that is by the ambiguity concerning the range of the variable "j". Does the teacher claim that the condition of surprise is only valid at the moment he utters it (j=1) or does he claim that it must be valid for the whole period (j=1 and j=2))?

In the former case his announcement is true because

${\sim}((d_1 \downarrow d_2) \supset d_1)$

that is

${\sim}(k° \supset d_1)$

satisfies (A16.xxix); therefore Eve's argument is illegitimate because the presumed incoherence of the announcement depends on

(A16.xxx) $S(d_2)$

which is not claimed by the teacher.

In the latter case Eve is right; in fact the announcement is incoherent because ((A16.xxx) is actually claimed, and

$(d_1 \downarrow d_2\& {\sim}d_1) \supset d_2$

that is

$k^1 \supset d_2$

contradicts ((A16.xxix).

A16.7.3. The extrapolation of the above analysis to n>2 days is only a matter of notational patience; the easy conclusion is that a precise agreement about

$S(d_n)$

can avoid any impasse. Furthermore the reason is explained why as days go by and the last possible date approaches, the surprise effect decreases (the number of partitioned alternatives is progressively reduced).

A16.8. Another violation of the competence condition involving the chronological dimension is instanced by usual statements like

((A16.xxxi) I believed that Jim was younger than he really was.

Of course ((A16.xxxi) is incoherent if interpreted as

at $t°$ I believed that Jim's age was less than itself

while the same ((A16.xxxi) is perfectly reasonable if interpreted as

at t' I have acquired information sufficient to correct
my previous belief about Jim's age

since the recognized existence of two chronologically different statutes legitimates the existence of incompatible beliefs.

A16.9. The most consequential violation of the competence condition is Frege's argument inducing him to claim that the content of a proposition cannot be its Bedeuting. Since the identity between Morningstar and Eveningstar does not hold in the statute of an individual who does not know it, the incidental difference in the alethic values of the two respective propositions is not evidence.