CHAPTER 15
VARIABLE

15.1. In §6 of *The principles of mathematics* Russell says that *variable* is one among the most difficult notions of Logic. From my informational viewpoint things are quite simpler: a free individual variable is a sign (a sign, I repeat) which refers to a precise member of the set constituting its domain without supplying sufficient information to single out the precise member it refers to. In other words a free variable is a sign affected by an institutional lack of information, and "the specific but here unspecified individual belonging to the domain of reference" or, roughly, "the individual to specify" are reasonable periphrases.

15.1.1. Of course even an indefinite article like "a" does not specify a precise member. Awaiting a detailed analysis, here I shortly note that the difference consists in the opposition *free* vs. *bond*, that is, for instance, the difference between *he* and *someone* (so to write, whatever he is someone, but only a particular someone is he).

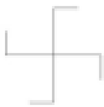
15.1.2. Therefore, though constants and variables are signs (syntactical entities), their discriminating factor is semantic. The distinction between pertinence and regard (§2.9.2) shows that defining an attribute concerning signs on the grounds of a semantic characteristic is a perfectly legitimate procedure. The (indeed rather consequential) opposition between impotent and active men is grounded on certain sexual faculties, but it concerns men, not sexual faculties. A variable, so to say, is a referentially impotent term.

15. 1.3. The current formal approaches conceal a lot of semantics behind *sign*. The semantic dimension is essential to something being a sign just as the matrimonial dimension is essential to someone being an ill-married man. Once castrated (I am speaking of the sign), that is once deprived of any meaning, it becomes a dead support of nothing more than itself, an arabesque devoid of any component able to make it a genuine sign.

15.2. I try to explain my viewpoint through a picturesque example. The individual constants of an emblematic language L^e are exactly (metalinguistic new-lines)

(15.i)

(which in the privileged interpretation designates Jesus),

(15.ii) 

(which in the privileged interpretation designates Hitler) and

(15.iii)

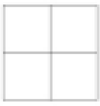
(which in the privileged interpretation designates De Gaulle). In L^e the graphically enlighting choice for the respective variable would then be

(15.iv)

because, since (15.iv) is the graphic core common to (15.i), (15.ii) and (15.iii), any constant is obtainable by (15.iv) through a graphic completion of this core-emblem; and the semantic integration saturating the lack of information adduced by the variable is nothing but the informational consequence of such a completion. In other words: three different fixations of (15.iv) lead to (15.i), (15.ii) and (15.iii) respectively, so determining three different semantic integrations.

Indeed, in general, settling a correspondence between graphic increments (decrements) of a sign and increments (decrements) of the adduced pieces of information would be a very precious convention. And I proposed L^e precisely because, except rare and ad hoc exceptions (for instance assuming "Ed" as a variable ranging over a domain whose members are Edgar, Edmond, Edward and Edwin) a convention like this is practically unrealizable in usual languages,

15.2.1. Obviously, since

(15.v) 

does not belong to L^e, the fixation from (15.iv) to (15.v) is senseless; in order to overcome this senselessness we should agree that in L^e (15.v) designates someone (Mondrian, say).

15.2.2. In §15.1 I emphasized that variables are signs. Particularly where variables are concerned, any ambiguity between the syntactic and the semantic planes is a calamity. Yet *variable*, in its current acceptation, is far from being an unambiguous notion: for instance in the same article of Wikipedia I read "a variable is a value …" and "a variable is used to designate a value…". And actually the contemporary orthodoxy considers as a variable both a variable quantity and a symbol (an expression) representing a variable quantity. But as "to be" and "to represent" are not synonyms, an ambiguity is consequently inevitable.

Before deepening this theme (§ 15.4), some (perhaps obvious) terminological agreement.

15.3. Variables range over domains (that is set of individuals). Sentences where at least one free variable occurs (free-variable-laden sentences) are open; otherwise (free-variable-free sentences) they are closed. The valence of an open sentence is the number of its different free variables. The proposition adduced by an open (closed) sentence is variant (invariant). The valence of a variant proposition is the valence of the open sentence it is adduced by. For instance

the river *x* is a border of state between *y* and *z*

is a trivalent sentence and

the Danube is a border of state between *y* and Bulgaria

is the monovalent sentence resulting from the fixation of *"x"* and *"z"*.

15.3.1. I spoke repeatedly of fixations. The syntactic component of a fixation is the replacement of a variable with a constant, that is, in L^e, the graphic integration transforming (15.iv) either in (15.i) or in (15.ii) or in (15.iii)). The informational component of a fixation is the filling up of the previous lack. A more punctual lexicon can distinguish between closure (operation on signs) and saturation (operation on pieces of information).

"Conversion" is another term I shall introduce (§16,4) to mean a peculiar fixation. Of course "substitution" (§15.9) is a less punctual term, since it can also refer to a case where a variable is replaced by a different variable, therefore to a case where no saturation is achieved.

15.4. *Variable* and *abstraction* are strictly related. With reference to L^e, just owing to the mentioned correspondence between graphic interventions on emblems and informational consequences, we can conceive (15.iv) as the result of an abstraction concerning the various constants, that is as the result of the operation through which we enucleate what the emblems (15.i), (15.ii) and (15.iii) have in common. In other words: if we neglect the peculiarities differentiating the single individuals of a certain set, we get a pattern that, covering the same path in the opposite direction, ramifies toward the various individuals of the domain. To abstract is to mutilate, to prune. In this sense, what Minsky (1989) calls "frame" can be called "variable".

Also *archetype* and *common noun* are notions strictly related to *variable*. The Snake, trivially, is not the hyper-uranic snake, but the approximative image we get from the attributes characterizing any real snake (a limbless et cetera animal); in this sense the Snake is nothing but the referent of the variable ranging over the set of snakes. But, exactly as the Personage (the referent of (15.iv)), the Snake is only the necessarily vague objectification of a necessarily vague mental pattern (obviously the more comprehensive is a pattern, the less detailed it is).

Where the domain of a variable, instead of being constituted by a set of well distinguishable individuals (as the personages above) is constituted by a variable quantity (the flow of this river), the separation among the various individuals of the domain is a matter of convention (the various flows) and as such it results less evident; so the whole phenomenon is spontaneously reduced to a diachronic identity (**the flow** of this river), that is to a variable quantity. Nevertheless the two contexts are not separated by any theoretically momentous factor. In both of them we have a variable (a sign) a domain et cetera; the difference is that in some contexts the values are similar enough to be assumed as instances of a same and diachronically mutable individual. And the example of §15.2 shows that the use of a variable is perfectly legitimate even where there is no variable quantity.

In other words. A variable is not characterized by a peculiarity (the variability) of its referent, but by a peculiarity (the institutional incompleteness) of the information adduced by the same variable. The introduction of

(15.iv) far from influencing the fact that Jesus, Hitler and De Gaulle keep on being the only members of our universe, influences our faculties of speaking about them, giving us the possibility to generalize our discourse by leaving the individual attributes out of consideration.

15.5. Another notion I wish to compare with *variable* is *(inclusive) disjunction*. Neither of them singles out anything, yet while ascribing

(15.vi) was a dictator

to the (inclusive) disjunction of (15.i), (15.ii) and (15.iii) adduces a true (then a closed) proposition because at least one of the three constants designates a dictator, ascribing (15.vi) to (15.iv) adduces a variant proposition, and as such a proposition to which no alethic value can be assigned. Hintikka (1973, I.5 footnote 26) refuses to read the quantifiers as disjunctions and conjunctions, yet my opinion is different; according to common sense I think that the piece of information adduced by the existential (or respectively the universal) quantification of a variable is the piece of information adduced by the inclusive disjunction (or respectively the conjunction) of its substitutors and that, therefore, no variance occurs. The actual root of the difference between

(15.vii) Jesus or Hitler or De Gaulle was a dictator

and

(15.viii) *x* was a dictator

(even when it is agreed that the same variable ranges over Jesus, Hitler and De Gaulle) is that while (15.vii) adduces *one of those personages* (then no blank), (15.viii) adduces *the specific but here unspecified personage* (then an intrinsic blank). In neither case is any personage singled out, but a disjunction adduces a piece of information which, so to say, is self-sufficient although it is less than the piece of information adduced by a precise constant (under the informational viewpoint, disjoining is decreasing).

In this sense I agree with Tarski's claim that a variable is something like the blanks of a questionnaire; this notwithstanding I cannot agree with his claim that variables have no specific meaning, since such a claim mistakes the meaning for the referent.

15.6. Also a proper name acts as a variable where there is a referential ambiguity. For instance, since in our party there are three fellows named "George", when I confide

(15.ix) George is in love

to Bob, he immediately asks me

George who?

("who?" "what?" et cetera are the classical questions revealing the presence of a free variable; and actually, in this context (15.ix) is an open sentence, "George" is a free variable,

All Georges (of this party) are married

is a true (then legitimate) quantification et cetera. On the other hand if I confide (15.ix) to the wife of one of the Georges, owing to the privileged role her husband plays (ought to play) in the mind of a wife, "George" becomes (ought to become) a constant designating a precise individual (her George, so to write). Here too the context is an important informational source; the use of a proper name with different referents in accordance with different interlocutors is a common practice, and actually (15.ix) is a closed sentence adducing three different propositions depending on which wife I address.

15.6.1. These simple considerations overcome the problem of individual descriptions where the uniqueness condition is not satisfied; such descriptions act as variables et cetera.

15.6.2. Of course referential ambiguity is not referential poorness. Bob, roughly speaking, has three rich images of his fellows named "George", but he does not know which of them is the one I am speaking of. On the contrary if I speak of Goedel with my mother-in-law, tacit reference is made to the only Goedel she knows, that is our enigmatic cat, while if I speak of Goedel with my barber, the only result is to put him in troubles (nearly as if I speak of Goedel with some logician I know).

15.7. The power of the informational approach is validated by its applicability to non-linguistic contexts, too. The theme will be carefully analyzed in next chapter: here I only propose a little example. The absolute silence of a crowded black-jack table is broken by the

(15.x) you are a cheat

the croupier utters staring at a precise gambler. While "you" is a constant for those who are looking at the croupier, since the direction of his eyes is the source of the informational integration necessary to single out the (presumed) cheat, the same pronoun is a variable (you who?) for those who are looking at the green baize and therefore do not perceive the stare. On the other hand if the croupier, instead of (15.x), says

(15.xi) here there is a cheat

he is quantifying ($\exists x(C(x))$) and the proposition adduced by (15.xi) is true or false simply because it is invariant.

15.8. While the connections relating *variable* with *parameter* and with *indeterminate* are well known and unproblematic, the distinction between *variable* and *unknown* needs a more punctual analysis. Though the "*x*" occurring properly in

(15.xii) *x* is odd

occurs properly and analogously in

(15.xiii) $20x + 177 = 13^3$

we spontaneously read (15.xii) as a propositional function (therefore its "*x*" as a very variable) and (15.xiii) as an equation (therefore its "*x*" as an unknown). Why?

Because of course (of course?) while we read (15.xii) as a formula adducing a proposition affected by an intrinsic lack of information (therefore as a formula adducing a variant proposition whose alethic assignation requires a previous saturation), we read (15.xiii) as a formula adducing a true proposition and therefore we read its "*x*" as a symbol used simply to mean that at first sight we are not in conditions to recognize what exact number it designates (what numeral it stands for).

The particular that while (15.xii) admits 'infinite' solutions, (15.xiii) admits only one, probably favours this discrepant readings; yet it is unquestionable that we can also read the "*x*" in (15.xii) as an unknown ($x = 1$ or $x = 3$ or et cetera) and the "*x*" in (15.xiii) as a variable, thus making (15.xiii) a propositional function (recycling use of a new line); in this case, obviously, only an 'external' fixation of the variable can transform the same (15.xiii) into an invariant proposition either true (for x=101) or false (otherwise).

The distinction between variables and unknowns is then reduced to the distinction between

the specific but here unspecified referent

and

the specific but here indirectly specified referent

and the similarity between the two periphrasis legitimates the use of similar symbols, so supporting the informational approach. In other words. The use of the same symbols both for variables and unknowns is justified by the common situation of interpretative perplexity; yet while the informational perplexity determined by an authentic free variable can only be overcome by an external supplement of information (the fixation), the informational perplexity determined by an unknown is overcome by the information adduced by the same unknown-laden sentence.

A certain misleading effect of the current terminology is that the unknown is (contextually) known. Anyhow I emphasize that, exactly as variables, unknowns too are signs, not 'quantities'.

15.9. The substitution of variables is a theme to treat in meticulous detail; particularly because the above stigmatized ambiguity affecting *variable* favours the diffused confusion between substitutors and values. For instance it is rather usual to read that if *"x"* is a numerical variable, then numbers are its values, or that if we substitute to a variable one of its values et cetera. The two expressions are radically incompatible; the (proper) values of a variable are the members of the set constituting the domain of the same variable, and its (proper) substitutors are the names of its (proper) values. Then, for instance, while the (proper) values of a numerical variable are numbers, its (proper) substitutors are numerals.

This simple distinction is very often neglected even by celebrated authors. The first instance in my hands involves Russell himself (*The Principles of Mathematics*, § 87): in order to legitimate a statement like

every value of a variable is then a constant

we must ambiguously agree that constants are both signs and represented values, so implicitly rejecting the unambiguous agreement according to which constants are signs (naming well specified referents).

15.10. Generally speaking, the main elements in a substitution are four, and precisely

- the situation ante substitution (I call it "initial configuration" or "base")
- what is replaced ("the substituendum")
- what takes its place ("the substitutor")
- the situation post substitution ("final configuration" or "resultant").

15.10.1. A substitution can be performed on very different situations and in accordance with very different operative modalities. For instance, let the banknotes in my wallet be the initial configuration, and let me consider the three substitutions where a 20$ banknote is replaced respectively by

(a) another \$20 banknote
(b) four \$5 banknotes
(c) one \$10 banknote.

Evidently while under (a) and (b) the initial and the final configurations have equal amounts, under (a) and (c) they are different; in order to mean this aspect I say concisely that while (a) and (b) are conservative, (c) is not. My statement is concise because, strictly, the notion of conservativeness is relative to a certain criterion (the purchasing power, in this case); in fact if the criterion were the number of banknotes in my wallet, (a) and (c) would be conservative and (b) would not. In this sense the correct expression must speak of conservativeness as regards a certain criterion; and preciselyt because when we deal with banknotes the main criterion is their purchasing power, if the specification is wanting we are induced to think of the economic value. I agree that "Ξ-conservative" abbreviates “conservative as regards the criterion Ξ”, and that

u’ = Subst(*u*° 20$/10$)

symbolizes the (c) substitution, being *u*° and *u*’ the initial and the final configuration of my wallet.

15.10.1.1. To speak of substitutions where the initial configuration is not modified (where the original $20 banknote is ‘substituted’ by the same banknote) would evidently be a terminological abuse; in fact “substitute” means *to put something in place of something else* and *else* is intrinsically incompatible with *same*. Just in this sense I remarked in §7.2.2 that “substitution of identity” is an oxymoron.

15.11. Logic is firstly interested in linguistic substitutions, that is in substitutions whose bases, substituends, substitutors and resultants are linguistic expressions.

In order to limit the analysis to the very problems themselves, I only deal with particular linguistic substitutions, and precisely with substitutions whose bases are syntactically and semantically proper atomic sentences (of course a sentence is syntactically proper iff it respects the well formation rules and semantically proper iff it is not affected by any sortal improperness). Furthermore the general formulation of an atomic sentence

$\mathbf{R}_n(u_1,\ldots, u_n)$

is epitomized in

P*(u)*

where **P** (without inverted commas, I am not speaking of the *ML*-symbol in boldface, but of an unspecified object predicate as, for instance, *“<(x+y-z)”*) is just an *n*–adic object relator et cetera.

Therefore a substitution (performed on a syntactically and semantically proper base)
- is syntactically proper iff substituends and substitutors have the same syntactical status
- is semantically proper iff substituends and substitutors are semantically homogeneous
- is proper iff it is syntactically and semantically proper.

So, for instance, let

V(b)

(Bob is vegetarian) be the (true or false, but anyhow proper) initial sentence. Then

Subst (“*V(b)*” *“b”/“V”)*

is

V(V)

that is a syntactically improper sentence. Analogously, once *“Q”* is assumed to symbolize “quadratic”,

Subst (“*V(b)*” *“V”/“Q”)*

is

Q(b)

that is a syntactically proper (both the substituend and the substitutor are adjectives and both of them belong to *L*), but a semantically improper sentence (the adduced attributes are heterogeneous, since *vegetarian* concerns living beings, while *quadratic* concerns numbers).

A quite peculiar kind of semantically improper substitutions are the projectively improper ones. For instance, since in

(15.xiv) Subst (“*V(b)*” *“b”/“ “b””)*

both the substitutend and the substitutor have the same syntactical status (both of them are terms), the substitution is syntactically proper (indeed

(15.xv) *V(“b”)*

is a syntactically proper sentence). This notwithstanding, since manifestly a name (“Bob”, in this case) cannot be vegetarian, (15.xv) is semantically improper. What makes it a projectively improper sentence (then what makes (15.xiv) a projectively improper substitution) is the peculiar link between the susbstituend and the substitutor, that is the name-relation). A projectively improper substitution occurs wherever the substitutor and the substitutend belong to different dialinguistic orders (in our case the substitutor belongs to the metalanguage of the language the substituend belongs to). The example (15.xiv) has been purposely chosen in order to make evident its projective improperness: while it is rather improbable to confuse a man with his name, it is much less improbable to confuse a name with its name (we shall see in due course how insidious is such a kind of improperness).

15 12. A (linguistic) substitution is *m*-conservative (meaning-conservative) iff its base and its resultant adduce the same piece of information (have the same meaning). So, for instance,

Subst (“Jim is cowardly” “cowardly”/”brave”)

is a proper yet a non-*m*-conservative substitution.

An *m*-conservative substitution, of course, is alethically conservative (ℵ-conservative); in fact, since alethic values concern pieces of information, an *m*-conservative substitution cannot modify the alethic value of the unmodified piece of information both the initial and the final configuration adduce.

This conclusion can be extrapolated as follows on the ground of the Theorems of Restriction and Expansion (§8.16). If the initial proposition is *k*-true and the substitutor adduces a piece of information implied by the piece of information adduced by the substituend, then the resulting proposition too is *k*-true. Concisely: a restrictive substitution is truth-conservative. Reciprocally, if the initial proposition is *k*-false and the substitutor adduces a piece of information implying the piece of information adduced by the substituend, then the resulting proposition too is *k*-false. Concisely: an expansive substitution is falsity-conservative. Of course an *m*-conservative substitution is both restrictive and expansive, therefore it is both truth-conservative and falsity-conservative (that is, ℵ-conservative).

15.13. In order to conform (provisionally) this text to the habitual terminology, in the subsequent sections truth and falsity will be directly ascribed to formulas and sentences. Therefore an expression like “the formula so and so is false” must be read as an abbreviation of “the proposition adduced by the formula so and so is false” or of “the formula so and so is fallacious”. For instance (metalinguistic new-line)

(15.xvi) Subst*(“x<f0” “x”/”ff0”)=”ff0<f0”*

Is a true ML-formula stating that if we substitute the *L*-constant *“ff0”* to the *L*-variable *“x”* occurring in the open *L*-formula *“x<0”* we obtain the closed *L*-formula *”ff0<f0”*. Indeed (15.xvi) is a closed *ML*-formula (no *ML*-variable occurs in (15.xvi) since in *ML* the name of any *L*-variable is a constant); and actually (15.xvi) is true because the described substitution actually yields the described resultant. Nevertheless (metalinguistic new-line)

(15.xvii) *ff*0<*f*0

is an evidently false *L*-formula. This alethic independence, owing to the (projective) difference between two dialinguistic orders, is absolutely unproblematic; in fact (let me insist)

- (15.xvi) speaks (in *ML*) of a relation between the operation on *L*-symbols described at first member and the *L*-formula at second member, then (15.xvi) is true because actually such an operation yields exactly the formula at second member;
- (15.xvii) speaks (in *L*) of a relation between numbers; then it is false because actually such a relation (to be less than) does not link the numbers that (15.xvii) speaks of.

Reciprocally

(15.xviii) Subst*(“x<f0” “x”/”ff0”)=”f0<ff0”*

is false because the described substitution does not yield the described resultant, which nevertheless is true because et cetera.

15.13.1. In general

Subst(**A v/n**)

is the substitution performed on an *L*-formula **A**, (here too without inverted commas, I am not speaking of the *ML*-symbol in boldface, but of an unspecified object formula as, for instance, *“x<f0”*) by replacing the variable **v** (as, for instance. *“x”*) with the numeral **n** (as, for instance “*ff*0”). The (sometimes very insidious) danger to avoid is mistaking a *ML*-variable like “**v**” (within inverted commas, now I am speaking of the symbol in boldface) with a *L*-variable like *“x”*; the former variable ranges over *L*-symbols, the latter over numbers). I wrote “sometimes very insidious” because peculiar conventions are possible (arithmetizations), under which both *ML*-variables and *L*-variables range over numbers, so masking a potentially pernicious projective improperness.

15.13.1.1. The relationship between *ML*-variables and *L*-variables is manifestly connected with the relation (§3.8.1 and §3.8.2) between *to abbreviate* (linking “Bob” with “Robert”) and *to denominate* (linking “Bob” with Bob). Since this passage is highly consequential, I will be pedantic.

Both the “*x*” occurring in (15.xii) and the “**n**” occurring in

n is a numeral

deal with numerals. Yet while “*x*” is an abbreviative variable which then **<u>stands for</u>** a numeral (so that any sentence obtained by erasing such variable and by writing a numeral in its place as for instance

*fff*0 is odd

is a proper sentence (speaking of the number 3, in the case), the latter is a designative variable, and as such, once erased, must be replaced by **<u>the name of</u>** a numeral; and in fact

“*fff*0” is a numeral

is a proper sentence (speaking of the expression naming the number 3).

In other words, numerals

- are the substitutors of *“x”* because they name numbers and the domain of *“x”* is just constituted by numbers:
- are the values of “**n**” because the domain of “**n**” is just constituted by numerals.

15.13.2. I take advantage of the occasion for a quite collateral (yet theoretically momentous) consideration. In the formal approach to arithmetic the names of the various numbers are not “0”, “1”, “2” et cetera, but “0” “*f*0”, “*ff*0” et cetera. Since what does vary is the number of “*f*”s concatenated to a final “0”, the adequate choice of the respective variables ought to substitute “*x*”, “*y*” et cetera with something like "ξ0", "ψ0" et cetera (of course this consideration concerns metamathematics, since it is evident that the usual choice is more economic for minute mathematical uses). The theoretical importance of the mentioned new variables is the possibility of a direct (non-recursive) definition of addition ($\xi 0+\psi 0=\xi\psi 0$); and the fact that, nevertheless, multiplication could not be defined directly, offers wide room of reflection about the respective consequences on the incompleteness (Presburger).

15.14. The choice of the variables occurring in a formula is an **absolutely crucial topic**. For the sake of concision, let me refer to numerical variables. Since they all adduce the same piece of information (roughly: *the number to specify*), the choice of *"x"* or *"y"* et cetera is arbitrary under many aspects. Yet as soon as the complexity of the formula increases, that is, as soon as the numbers to specify are more than one, the choice is conditioned by the necessity to respect the identity and non-identity among the various unspecified numbers we are speaking of through the identity and non-identity among the respective variables. Therefore in order to avoid undue interferences and highly misleading conclusions our choice must comply with the two following rules:

R1: not to choose the same variable for non-necessarily identical numbers (or, worse, for necessarily non-identical numbers)

R2: not to choose different variables for necessarily identical numbers.

15.14.1. The two rules are nearly obvious, yet I support them by some example. Since both

(15.xix) $x/y=z$

and

(15.xx) $x/w=z$

say generically that a certain number is equal to the ratio of two others, (15.xix) and (15.xx) are interchangeable formulations. On the other hand

(15.xxi) $(x=yz) \supset (x/y=z)$

symbolizes correctly an elementary mathematical truth, and (15.xix) is the apodosis of (15.xxi); this notwithstanding the substitution of (15.xx) to (15.xix) in such an apodosis would lead to

(15.xxii) $(x=yz) \supset (x/w=z)$

and (15.xxii) is an evidently unacceptable formula because "*w*" does not witness the necessary identity with the number indicated by "*y*" in the protasis, that is because **R2** is violated. Reciprocally

(15.xxiii) $(x=yz) \supset (x/y=y)$

is an evidently unacceptable formula because the third occurrence of "*y*" in the apodosis of (15.xxiii) entails a false identity between the divisor and the result of the division, that is because **R1** is violated.

Let me insist through a comprehensive example. While

$\exists u(u<y)$

and

$\exists u(u<x)$

are interchangeable formulations

$(x+y \neq x) \supset \exists u(u<y)$

is a true conditional, but

(15.xxiv) $(x+y \neq x) \supset \exists u(u<x)$

is false, exactly because the last occurrence of *"x"* violates both **R1** and **R2**; in fact, with reference to the last variable occurring in (15.xxiv), whatever choice different from *"y"* violates **R1** because it does not witness an actual identity with the *"y"* occurring in the protasis, moreover the choice of *"x"* violates **R2**, too, because it entails an abusive identity with the "*x*" already occurring in the protasis.

15.15. The well known condition ruling the substitutions where variables occur (the substitutor must be free for the substituendum) is usually justified by an empirical procedure, that is by exhibiting examples where some absurdity is derived from the violation of the mentioned condition (cf. for instance Kleene 1971; §18 Example 8; Hermes 1973, IX §1; Shoenfield 1967 §2.4). The informational approach allows a theoretical explanation, just by remembering the necessity to avoid undue identifications. In fact if the substituendum occurs free in the scope of a quantifier concerning a variable occurring in the substitutor, then the substitution would violate **R2** by establishing an undue identity (that is an identity absent in the initial formula). The formal procedure to avoid such a risk, that is the re-denomination of the bound variables (Hermes, ibidem IX §2) is simply the way to restore the respect of **R2.**

The fundamental conclusion is that a substitution may restrict the arbitrariness in the choice of variables.

15.16. The informational approach to *variable* allows also the derivation of

(15.xxv) $\exists x P(x)$

from

(15.xxvi) $P(a)$

and the derivation of (15.xxvi) from

(15.xxvii) $\forall x P(x)$

because (15.xxv) is a restriction of (15.xxvi) and (15.xxvi) is a restriction of (15.xxvii), therefore the truth of (15.xxvi) implies the truth of (15.xxv) and the truth of (15.xxvii) implies the truth of (15.xxvi).

An analogous achievement is §17.5.2.