CHAPTER 14
CONDITIONALS

14.1. I call generically “conditional” the propositional connection

(14.i) if *p* (then) *q*

where *p* and *q* are respectively the protasis (or antecedent) and the apodosis (or consequent).

Nowadays, as far as I know, conditionals are canonically theorized through a truth functional approach. Since this approach seems to me fragmentary and inadequate, in this chapter I propose a systematic alternative. The formalization of such an alternative leads to a rather complex paradigm; yet far from being a fault, such a complexity is quite favourable evidence. In fact (let me repeat the consideration proposed in §8.15.3), since all its voices can be supported by punctual and non-interchangeable examples, a simpler paradigm would only mean too rough a classification.

For the sake of peace I will do my best in order to adapt my terminology to the current one (and “truth functional” is an example). And for the sake of concision, for instance, I will write

(14.ii) if even (then) <6

in order to mean that if a certain outcome is even then it is less than 6.

14.2. Let “Д” be a variable on propositional connectives, “$\aleph$” a variable on alethic values, “*x*” and “*y*” (with or without indexes) variables on propositions. An *n*-adic propositional connective

$\text{Д}(x_1 \,....\, x_n)$

is a truth function iff the alethic value $\aleph(\text{Д}(x_1 \,....,\, x_n))$ is deducible from $\aleph(x_1) \,...\, \aleph(x_n)$. Therefore if $\text{Д}(x_1, \,....,\, x_n)$ is a truth- function, its alethic value does not change if we replace x_i with an y_i such that $\aleph(x_i)=\aleph(y_i)$.

14.3. Let

- conjunction is a truth function
- negation is a truth function
- the conjunction of truth functions is a truth function
- the negation of truth functions is a truth function
- the only truth functions are those given above

be the recursive definition of truth functions.

Therefore

(14.iii) $\sim(p\&\sim q)$

is a truth function (false iff *p* is true and *q* is false).

14.4. Reading (14.iii) as (14.i) gives rise to well known conclusions in frontal contrast with our common sense.

Bob bet on 6, but the outcome was 3. I comment

If the outcome had been 2, you would have won

and I reply to his astonished glance that until we agree to read (14.iii) as (14.i), every outcome different from 3 implies the truth of my comment (obviously a conjunction where a false statement occurs is anyhow false, and as such its negation is anyhow true).

His new astonished glance reveals that, in his opinion, logic is a perhaps esoteric but surely crazy doctrine totally incompatible with his most deep-seated convictions.

Orthodox logicians neglect these difficulties; they claim that the task of the truth functional approach is to present an alethic table for each connective, not to justify consequences entailed by reading the same connectives in a certain way. Yet their attitude eludes the very hearth of the problem, that is giving a trustworthy theoretic frame to the logic by which the current use of “if (then)” is ruled. Surely this logic does exist, since in current practice we are able to distinguish sensible and senseless conditionals. And the problem too exists: why, for instance, does reading

$\sim p\&\sim q$

as

neither *p* nor *q*

not create any impasse, while reading (14.iii) as (14.i) does?

My answer is that the logic of *if (then)* (exactly as the logic of *because* or of *though*) cannot be theorized through a truth functional approach, but, once more, through an informational approach. According to this approach the soundness of a conditional depends on some link between the pieces of information respectively adduced by its protasis and its apodosis. And the conclusions we reach agree perfectly with our common sense.

14.5. For the present (until §14.15) let me reason on the possibility space (Howson & Urbach 2006, §2.a) or universal set (Hajek 2003, §1) **Ω** constituted by the outcomes of a die. In particular while **Ω°** is the possibility space related to a standard (cubic) die, **Ω'** is the possibility space related to a dodecahedral die. Therefore, since every outcome must present one and only one number, the possible outcomes are connected by partitive disjunctions (XOR), so that

(14.iv) either 1 or 2 or 3 or 4 or 5 or 6

is the basic statute *k°* for **Ω°**. just as

(14.iv') either 1 or 2 or 3 or 4 or 5 or 6 or 7 or 8 or 9 or 10 or 11 or 12

is the basic statute *k'* for **Ω'**.

14.6. Statutes play a fundamental role in the informational approach to conditionals. Generally speaking, to agree that we are reasoning under a certain statute is to agree that the pieces of information constituting such a statute enter into the knowledge we use in order to state our conditionals.

For instance, again with reference to the rolling of a die, if we assume that all we know about a certain outcome is

(14.v) Bob bet on 6 and lost

then (14.ii) is true with reference to *k°*, but false with reference to *k'* where also 8, 10 and 12 are possible even (yet ~<6) outcomes. This dependence of the truth value on the statute of reference shows that, strictly, statements like

(14.vi) (14.ii) is true

are elliptical.

Let me insist: (14.vi) is elliptical because, given (14.v), (14.ii) is *k°*–true and *k'*–false. In the current practice we can unequivocally use elliptical conditionals only where the statute of reference is unequivocally established by the specific context (for instance when we simply speak of a die, our interlocutor is induced to think of a cubic one, not of a dodecahedral one). But of course in a formal approach we must make explicit reference to the statute, because its pieces of information are a necessary component of the implicative relations on whose grounds the paradigm is structured.

14.7. The informational approach leads to a paradigm where the various kinds of conditionals are classified on the grounds of implicative relations among the three protagonists of any conditional, that is its protasis *p*, its apodosis *q* and its statute of reference *k*. Yet, in order to simplify the formulae of the following theorization, the statute of reference is omitted (that is: it is given as implicitly understood).

First of all let me underline two preliminary assumptions.

14.7.1. The assumption of coherence

(14.vii) $\sim(p\supset\sim q)$

avoids wasting time about conditionals like

if even, then 3

whose absurdity is immediately evident owing to the logical incompatibility between protasis and apodosis.

Without the agreed simplification concerning the omission of the statute, of course,

(14.viii) $\sim(k\&p\supset\sim q)$

should replace (14.vii). Yet such a simplification is confirmed for every formula till (14.xxxii).

14.7.2. The assumption of properness is grounded on the distinction opposing proper conditionals to spurious (improper) ones. A conditional is proper iff both its protasis and its apodosis adduce information about the same possibility space. In ® such an assumption imposes that the sectors representing the protasis and the apodosis concern the partition of a same circle.

So, normally,

(14.ix) if the croupier is a polygamist, then ~3

is a spurious conditional because, normally, his polygamy does not influence the outcomes of the die he rolls. Yet if we reason within a (whimsical) context where

- there is only one polygamous croupier
- the polygamous croupier is an ill-famed trickster
- the trick of the die he rolls forbids the outcome 3

then (14.ix) is a proper (and true) conditional; in fact under such a (whimsical) context to learn that the croupier is the polygamist is to acquire a piece of information about the possible outcomes of the die, that is a piece of information which increases the statute by erasing "or 3" from (14.iv) (or from (14.iv') if the rolled die were dodecahedral). Anyhow, according to the properness assumption we neglect spurious conditionals and whimsical contexts: only proper conditionals are the objects of the following theorization.

14.8. The piece of information *c* such that

(14.x) $p\&c \supset q$

and
(14.xi) $p\&\sim q \supset \sim c$
is called "**content** (of the conditional)". Informally speaking, *c* is the smallest piece of information which (under the statute of reference) must be added to the protasis in order to imply the apodosis. In other words, (14.x) and (14.xi) are rules that every kind of conditional must satisfy.

Let me emphasize that the content *c* depends not only on the protasis *p* and on the apodosis *q*, but also on the statute *k*. For instance, under our *k*°, the content of
(14.xii) if even, then 6
is
(14.xiii) ~2&~4
because (14.xiii) is the smallest piece of information which, added to (14.iv) and to the respective protasis
~1&~3&~5
implies exactly
6
that is the apodosis. But under *k'* the content of (14.xii) would be
(14.xiii') ~2&~4&~8&~10&~12
because (14.xiii') is exactly the smallest piece of information which must be added to the conjunction of the statute (14.iv') and to the respective protasis
~1&~3&~5&~7&~9&~11
in order to imply the apodosis. Henceforth the examples neglect *k'* and make (implicit) reference to *k*°.

14.8.1. The insertion in *c* of superfluous elements is precluded by (14.xi). For instance
(14.xiv) ~2 & ~4 and the die is red
cannot be the content of (14.xii) because
(14.xv) ~(~2 & ~4 and the die is red)
is the opposite of (14.xiv), but evidently while $p\&\sim q$, that is
even & ~6
does imply the opposite of (14.xiii), that is
~(~2&~4)
it does not imply (14.xv) because it does not tell us anything about the colour of the die.

14.9. My central claim is that the alethic value of a conditional is the alethic value of its content. This claim can be informally but easily supported by ascertaining that, after all, the content of a conditional is the piece of information the conditional in question transmits us.

Many kinds of conditionals can be distinguished. Their classification is centred on the informational relations among *p*, ~*p*, *q* and ~*q*.

The conditionals where
(14.xvi) $p\supset q$
are called "implications".

The conditionals where
(14.xvii) $\sim p\supset q$
are called "para-enthymemes".

No conditional can be at the same time an implication and a para-enthymeme: if it were, by *Modus Tollens* we could derive from (14.xvi) and 14.xvii)
(14.xviii) $\sim q\supset(p\& \sim p)$
that is an incoherence.

The conditionals where
(14.xix) $\sim(p\supset q) \& \sim(\sim p\supset q)$
are called "enthymemes". Evidently, since (14.xix) refuses both (14.xvi) and (14.xvii), no enthymeme can be an implication or a para-enthymeme; therefore the set of proper conditionals is partitioned (no blank, no overlap) by the three mentioned kinds of conditionals.

In particular the enthymemes where
(14.xìx) $\sim(\sim p \supset\sim q)$
are called "weak (enthymemes)", and their opposites, that is the enthymemes where
(14.xxi) $\sim p \supset\sim q$
are called "strong (enthymemes)".

Of course every kind of conditional must possess all the respective requisites. So for instance the complete rule for weak enthymemes is
(14.xxii) $\sim(p\supset\sim q) \& \sim(p\supset q) \& \sim(\sim p\supset q) \& \sim(\sim p\supset\sim q))$

stating respectively that the conditional under scrutiny is coherent, and that it is neither an implication nor a para-enthymeme nor a strong enthymeme. Analogously

(14.xxiii) $\sim(p\supset\sim q)\ \&\ \sim(p\supset q)\ \&\ \sim(\sim p\supset q)\ \&\ (\sim p\supset\sim q)$

is the complete rule for strong enthymemes.

Now let me analyse the classification.

14.10. Implications are a rather trivial topic. A collation between (14.x) and (14.xvi) shows immediately that an implication is a conditional with a null content. For instance

(14.xxiv) if even, then >1

satisfies (14.xvi), and indeed (14.iv) tells us that no further information is needed to conclude that (14.xxiv) is true.

14.10.1. To claim that conditionals are not truth functions is to claim that implications are not truth functions. In fact, since negations and conjunctions are truth functions and since (§ 14.9) the various kinds of conditionals are defined on the sole basis of negations, conjunctions and implications, if implications were truth functions, the recursive definition of §3 would entail that conditionals too are truth functions. Let me propose a little example validating this conclusion. Although it is true that Germany is in Europe (*g*), that Berlin is in Germany (*b*), that Kyoto is in Japan (*j*) and that Berlin is in Europe (*e*),

$b\&g \supset e$

is true, but

$b\&g\supset j$

is false (contrary to *Berlin is in Europe*, *Kyoto is in Japan* is not deducible from *Berlin is in Germany* and *Germany is in Europe*); therefore both true and false implications exist whose protasis and apodosis are true.

This conclusion is perfectly compatible with the informational approach, where an implication is valid (under a certain statute) iff the piece of information adduced by its apodosis is deducible (under the same statute) from the piece of information adduced by its protasis.

14.10.2. Conditionals like

(14.xxv) if *p* then even or odd

(that is conditionals whose apodosis is necessarily true) could be read as limit implications.

14.10.3. Conditionals like

if even, then even

instance a banal sort of implication where $p=q$ (tautological conditional).

14.10.4. The existence of implications like

if the outcome is >2 and not divisible by 3, it is either quadratic or prime

whose understanding is not so immediate, is of no theoretical moment.

14.11. Para-enthymemes are a strained kind of conditionals where the same “if” becomes a potentially misleading occurrence, since it seems to subordinate the apodosis to a condition which such an apodosis is not at all subordinate to. For instance

(14.xxvi) if even, then ~2&~4

is an unquestionable para-enthymeme because

(14.xxvii) if odd, then ~2&~4

is an unquestionable implication. The formal proof that in a para-enthymeme (without any appeal to *p*) *c* is sufficient to imply *q* runs (concisely) as follows:

(1)	$p\&c\&q = p\&c$	(from (14.x))
(2)	$p\&c\&\sim q = \perp$	(Chapter 7, Theor 10)
(3)	$\sim p\supset q$	(rule for para-enthymemes)
(4)	$\sim p\&q = \sim p$	(from (3), by definition)
(5)	$\sim p\&\sim q = \perp$	(Chapter 7, Theor 10)
(6)	$\sim q\&\sim p = \perp$	(Chapter 7, AX4)
(7)	$\sim q\&p = \sim q$	(Chapter 7, Theor11)
(8)	$c\&\sim q = \perp$	(substitution of identity (7) in (2))
(9)	$c\&q = c$	(Chapter 7, Theor11)

u.d.e.

14.11.1. Conditionals like

if even or odd, then *q*

(that is conditionals whose protasis is necessarily true) could be read as limit para-enthymemes, since the basic requisite is kept according to which the protasis does not give any informational contribute. Indeed also conditionals like (14.xxv) can be read as para-enthymemes, since their apodosis holds quite independently on the protasis. In spite of this double reading (as limit implication and limit para-enthymemes), the specific context makes (14.xviii) an acceptable consequence, because $\sim q$ becomes $\bot$.

14.11.1.1. Conditionals like

if even or odd, then <7

(that is conditionals whose protasis and apodosis are necessarily true) sound as a rhetorical way to emphasize the necessary truth of the apodosis by the necessary truth of the protasis.

14.12. Enthymemes are the core of conditionals, exactly because both protasis and content play a non superfluous role. First of all I propose two examples where (always under our statute $k°$), the same protasis (if even) and the same content (~2&~4) lead to different enthymemes owing to their different apodoses.

14.12.1. The conditional

(14.xxviii) if even, then >4

is a weak enthymeme. In fact, as

- being even does not imply being >4 (for instance 2 is even and ~>4)
- being even does not imply being ~>4 (for instance 6 is even and >4)
- being odd does not imply being >4 (for instance 3 is odd and ~>4)
- being odd does not imply being ~>4 (for instance 5 is odd and >4)

the complete rule (14.xxii) is satisfied by (14.xxviii).

14.12.2. The already proposed conditional (14.xii), that is

(14.xii) if even, then 6

is a strong enthymeme. In fact, as

- being even does not imply being 6 (for instance 4 is even and ~6)
- being even does not imply being ~6 (for instance 6 is even and ~(~6))
- being odd does not imply being 6 (for instance 3 is odd and ~6)
- being odd implies being ~6 (no odd number can be 6)

the complete rule (14.xxiii) is satisfied by (14.xii).

14.12.3. The passage from a weak enthymeme to a strong one having the same content, is legitimated by a variation of the informational endowment entailing the refusal of q when $\sim p$. And such a variation can be attained through two reciprocal paths, that is either through an increment of q or through a decrement of p. For instance, in order to transform (14.xxviii) into (14.xii), we can

- either to follow the first path by adding ~5 to its apodosis >4 (the increased apodosis >4&~5 is exactly 6);
- or to follow the second path by ablating the same ~5 from its protasis which thus becomes ~1&~3 (evidently

if ~1&~3, then >4

is a strong enthymeme because the opposite of its protasis (~(~1&~3), that is 1∨3) implies the opposite of the apodosis (~ >4)).

14.13. The diagrams of ® can efficaciously help the intuitive understanding of the classification. Of course $k°$ results from the uniform partition of the circle in six sectors. So, while Figures 14.1 represents p, (odd sectors precluded) and Figure 14.2 represents $\sim p$ (even sectors precluded)

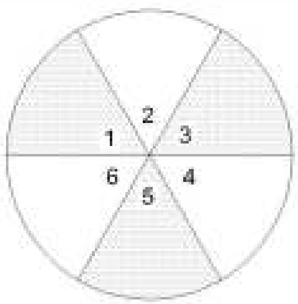

Figure 14.1

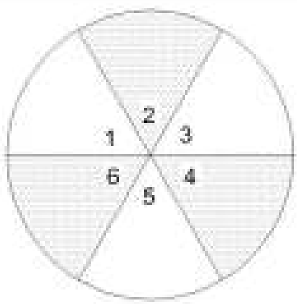

Figure 14.2

Figure 14.3 and Figure 14.4

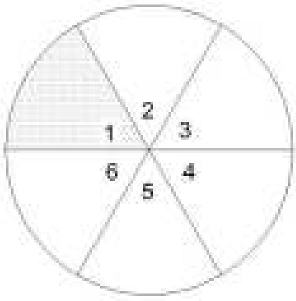

Figure 14.3

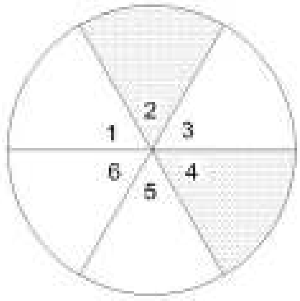

Figure 14.4

represent respectively >1, and ~2&~4, that is respectively the apodoses of the implication (14.xxiv) and of the para-enthymeme (14.xxvi), exactly as Figure 14.5 and Figure 14.6

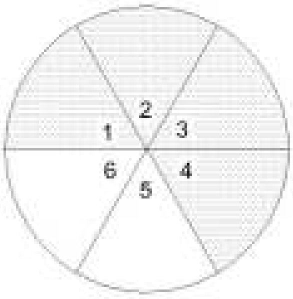

Figure 14.5

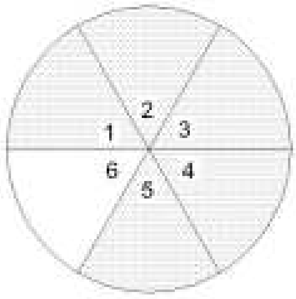

Figure 14.6

represent respectively >4 and 6, that is respectively the apodoses of the weak enthymeme (14.xxviii) and of the strong enthymeme (14.xii).

On the ground of what I affirmed in §14.8 ("*c* is the smallest piece of information which must be added to the protasis in order to imply the apodosis") such a content is represented by shading in Figure 14.1 all the sectors necessary to make the shaded field of the apodosis under scrutiny completely covered by the shaded field of the new diagram. Therefore the collation between Figure 14.1 and Figure 14,3 shows that no sector must be shaded (null content of an implication) in order to achieve such a result: Analogously a collation between the pairs of the other three conditionals shows that their content is the same, since in all of them it is represented by shading the sectors 2 and 4 (that is: such a content is in any case ~2&~4).

All the conclusions of the above analysis are punctually verifiable through the figures. Two examples.

14.13.1. According to the definition (14.xvii), (14.xxvi) is a para-enthymeme because (14.xxvii) is an implication. Such evidence is represented by the fact that the shading field of Figure 14.2 covers the shading field of Figure 14.4.

14.13.2. The diagram obtained by representing also the content, that is by shadings also sectors 2 and 4 of Figure 14.1 is exactly Figure 14.6; this means that the apodosis of the strong enthymeme (14.xii) adduces precisely the piece of information resulting from *p&c*. The argument proposed in §14.12.3 results from a collation between Figure 14.5 and Figure 14.6, whose only difference concerns sector 5.

14,13.3. The representation of a spurious conditional would need two circles, one for the possibility space of the protasis and one for the possibility space of the apodosis. The whimsical context of §14.7.2 allows us to shade sector 3 as a consequence of the croupier's polygamy; otherwise such a sector is virgin. In this sense spurious conditionals might be considered as a degenerate form of para-enthymemes (§14.11.1.1).

14.14. The definitions (14.xvii) (14.xx) and (14.xxi) show that the answer to the crucial question

(14.xxix) and if ~*p*?

is the essential factor ruling not only the distinction between para-enthymemes and enthymemes, but also the distinction between strong and weak enthymemes. In fact the answer to (14.xxix) must be

(14.xxx) then *q*

if the conditional is a para-enthymeme (as for instance (14.xxvi)),

(14.xxxi) then may be *q*, may be ~*q*

if the enthymeme is weak (as for instance (14.xxviii)), and
(14.xxxii) then $\sim q$
if the conditional is a strong enthymeme (as for instance (14.xii)).

14.15. Until now I reasoned on the extremely simple possibility space represented by the outcomes of a die. Yet in our minute practice we mainly use conditionals with reference to highly complex possibility spaces concerning the world we live in. In order to acknowledge the deep differences between conditionals like the above instanced ones and conditionals like
(14.xxxiii) if Jim wins the lottery prize, he will purchase a country seat
I speak respectively of analytic (or unambiguous) and of synthetic (or ambiguous) conditionals.

Since an explicit protasis and an explicit apodosis occur in synthetic conditionals too, their ambiguity depends only on their statute (and, consequently, on their content). This means, following the example, that the same (14.xxxiii) which under some statute may be read as a para-enthymeme under another statute may be read as a weak or as a strong enthymeme. In other words, a univocal classification of (14.xxxiii) may be hampered or even forbidden by the lack of more specific information about the context, that is by the impossibility of establishing a univocal statute (and, consequently, a univocal content).

The necessity of establishing a precise statute of reference in order to derive a precise content, then a precise alethic value, entails the necessity of replacing the simplified rule (14.vii) with (14.viii) and analogously the necessity of replacing (14.x) and (14.xi) with
(14.xxxiv) $k\&p\&c \supset q$
and
(14.xxxv) $k\&p\&\sim q \supset \sim c$
respectively.

14.15.1. There are many (and potentially contrasting) reasons that can induce Jim to purchase a country seat if he wins the lottery prize: the social ambition (only the owners of a country seat can ...), a damnable promise to his wife (I promise you that if I win ...), a mere financial choice (today the best investment is ...) et cetera. Yet, since (14.xxxiii) does not inform us about such reasons, I will speak of a firm intention to summarize generically all of them.

Once obviously assumed
- Jim wins the lottery prize
as *p* and
- Jim purchases a country seat
as *q*, let me schematically assume
(14.xxxvi) - the lottery prize gives sufficient suitable funds for purchasing a country seat
- sufficient suitable funds and the firm intention to purchase entail a purchase
as *k*, and
(14.xxxvii) - Jim firmly intends to purchase a country-seat
as *c*. Under these assumptions, since *q* (Jim purchases a country seat) is actually deducible from $k\&p\&c$ in conformity with (14.xxxiv), no further information is necessary to make true (14.xxxiii). Furthermore, since (14.xxxv) too is satisfied, (the lottery gave Jim sufficient suitable funds, nevertheless he did not make the purchase, then he had not the firm intention), the content of (14.xxxiii) is actually (14.xxxvii). Yet which kind of conditional (14.xxxiii) is? In compliance with (14.xxix), that is
and if Jim does not win?
is the basic question whose answer allows the classification of (14.xxxiii). And such an answer depends on the pieces of information we add to (14.xxxvi). I sketch three possible contexts.

Under k_1 Jim, besides being a very rich person who recently inherited a lot of gold bars (thus disposing of sufficient suitable funds) is a hard speculator who thinks that today the purchase of a country seat is the best investment and an ambitious man aspiring at the admission to an illustrious club where only important land owners are welcome. Under this k_1, (14.xxxiii) tends to be read as a para-enthymeme (Jim will purchase anyway).

Under k_2, Jim, besides being a very rich person who recently inherited a lot of gold bars, is an absolutely unforeseeable guy: what then will he decide about the purchase if he does not win? Under this k_2, (14.xxxiii) tends to be read as a weak enthymeme.

Under k_3, Jim is a poor and honest pensioner with no chance of borrowing money or of robbing a bank et cetera. Under this k_3, (14.xxxiii) tends to be read as a strong enthymeme.

The point to remark is that the considerations about the alethic value of (14.xxxiii), which depends on the alethic value of its respective content are quite distinct from the considerations about the alethic value of the protasis, which depends on the result of the lottery.

14.15.2. Of course the analysis above is far from exhausting the informational nuances offered by the huge complexity of the statute in which the conditional under scrutiny is inserted. For instance, let (14.xxxiii) be a gossip

Bob imparts to me; since both of us know that while Jim repeatedly declared his intention to purchase a yacht if should he win the lottery prize, Jim's wife repeatedly declared her intention to purchase a country seat if should she win the lottery prize, in such a situation the content of (14.xxxiii) is something like *Jim is dominated by his wife*.

On the contrary, if we know that while Jim's wife repeatedly declared her intention to purchase a yacht if should she win the lottery prize, Jim (who cordially detests the country life) repeatedly lamented that his condition of a without property guy forbids his admission to the mentioned club, the content of (14.xxxiii) is something like *Jim is dominated by his social ambition*. A more sophisticated analysis could be carried out by supposing that only Bob knows the context, so that his statute and mine (therefore his content and mine) are different et cetera.

A last example follows from the hypothesis that (14.xxxiii) is a statement I hear casually, without knowing Jim; under such a hypothesis the absolute ambiguity of the respective statute (the disjunction of too many alternatives, each of them with its respective content) forbids any alethic assignation and any classification.

14.16. The truth functional connection (14.iii), that is, more generally (§7.11),

(14.xxxviii) $\sim(h_1\&\sim h_2)$

is called "pseudo-hypothetic". While an inference from the proper conditional

(14.xxxix) if h_1, then h_2

to the corresponding pseudo-hypothetic is always legitimate, the reciprocal inference may be illegitimate. In fact while the truth of (14.xxxix), besides assuring the truth of (14.xxxviii), that is the falsity of

(14.xxxx) $h_1\&\sim h_2$

assures also that h_1 and h_2 are two pieces of information concerning the same possibility space, no properness condition binds the truth of (14.xxxviii), that is the falsity of (14.xxxx), therefore the properness of (14.xxxix) is not assured.

For instance, the truth of (14.xxxiii) entails that

Jim wins and he does not make the purchase

is false; but the falsity of

Jim wins and Kyoto is not in Japan

does not entail that

If Jim wins, then Kyoto is in Japan

is a proper conditional. Connecting two spurious statements by "if (then)" is a misleading procedure.

14.17. As to composite conditionals like

(14.xxxxi) $p_1\supset(p_2\supset q)$

it is easy to ascertain that (14.xxxxi) and

(14.xxxxii) $(p_1\&p_2)\supset q$

are equivalent; in fact

$(p_1\&p_2\&q)=(p_1\&p_2)$

can be indifferently deduced by definition from (14.xxxxii) or by substitution of identity ($p_2\&q=p_2$) from (14.xxxxi).

14.18. Another traditional theme disqualifying the truth functional approach to conditionals (Quine 1959, §§ 2 and 3) concerns counterfactuals (*i.e.* conditionals whose protases are openly assumed as false). In my opinion an approach entailing that an outcome 3 makes true both

(14.xxxxiii) if the outcome had been 2, Bob would have won

and

(14.xxxxiv) if the outcome had been 2, Bob would have lost

is to refuse without appeal, particularly because the informational approach theorizes counterfactuals in a way which complies perfectly with our intuitive suggestions. Briefly, I explain myself by comparing

(14.xxxxv) if the outcome is 2, Bob wins

with (14.xxxxiii). The difference between moods and tenses of their verbs means just two different positions about the alethic value of the protasis (false in the counterfactual (14.xxxxiii), unknown in the indicative conditional (14.xxxxv)). Yet their content is the same (*Bob bet on 2*). Therefore, since in the informational approach the alethic value of a conditional is the alethic value of its content, if (14.xxxxiii) is true (14.xxxxv) too is true, and if (14.xxxxiii) is false (14.xxxxv) too is false. Reciprocally, since the content of (14.xxxxiv) is the exact opposite (*Bob did not bet on 2*), if (14.xxxxiii) is true (14.xxxxiv) is false, and if (14.xxxxiii) is false, (14.xxxxiv) is true.

In other words. The transformation of an indicative conditional into the corresponding counterfactual (or vice versa) modifies the adduced pieces of information only as for the alethic value of the protasis (perhaps true or perhaps false in the indicative conditional, surely false in the counterfactual); yet, since such a transformation does not modify their common content, it does not modify their common alethic value.

14.19. Indeed it seems to me that all these conclusions agree perfectly with our intuition.